SER of Uncoded OFDM Systems with Insufficient Guard Interval Length and on Time-Varying Channels

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Abstract— **We address the impact of a too short guard interval (GI) length and time variations of the channel on the symbol error ratio (SER) of an OFDM system. Provided that the interference powers caused by these two effects are equal, their impact on the SER of the system are also identical. Both analytical and simulation results illustrate our claim.**

Keywords— **OFDM, interference and symbol error ratio analysis.**

I. Introduction

In OFDM systems, it is well known that the effect of time variations of the channel causes intercarrier interference to the system. The effect of insufficient guard interval length causes both intercarrier and intersymbol interference. The interference powers caused by the first and second mentioned effects above are exactly obtained in [4] [5] [8]. Now, it is interesting to see the influences of each of these effects on the symbol error ratio of the system.

In this paper, section II presents the theoretical results of the useful power for an OFDM system in the case of insufficient guard interval length and on a time-varying channel. This result can be used to calculate the total interference power of the system. Section III shows how we can evaluate the SER of an OFDM system with insufficient guard interval length and on a Rayleigh fading channel. Finally, simulation and theoretical results are compared in section IV.

II. Mathematical description of useful and interference power

Similar to [4], carrier and timing synchronization are assumed to be perfect and all analyses are considered in baseband. In the case of insufficient guard interval and time-varying channel, the demodulated symbol $\hat{d}_{l,i}$ on the l-th sub-carrier and the i-th OFDM symbol after taking the Fourier transform is given [4] as follows:

$$\hat{d}_{l,i} = \frac{1}{T_{\mathrm{S}}} \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \left\{ \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i} \int_{\tau=0}^{\tau_{\max}} h(\tau,t) g(t-\tau-iT'_{\mathrm{S}}) \cdot e^{-j2\pi n f_{\mathrm{s}}\tau} d\tau \right\} e^{j2\pi(n-l)f_{\mathrm{s}}(t-iT'_{\mathrm{S}})} dt$$

$$+\frac{1}{T_{\mathrm{S}}} \sum_{i'=-\infty, i'\neq i}^{+\infty} \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \left\{ \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i'} \int_{\tau=0}^{\tau_{\max}} h(\tau,t) \cdot g(t-\tau-i'T'_{\mathrm{S}}) e^{-j2\pi n f_{\mathrm{s}}\tau} d\tau \right\} \cdot e^{j2\pi f_{\mathrm{s}}[n(t-i'T'_{\mathrm{S}})-l(t-iT'_{\mathrm{S}})]} dt, \quad (1)$$

where T_{S}, T'_{S}, N_{C} are the OFDM symbol duration, the OFDM symbol duration plus GI, and the number of sub-carriers, respectively. The subscripts n, l denote the sub-carrier index, and i, i' represent the OFDM symbol index. $g(t)$ is the basic impulse of all sub-carriers defined in [4]. $f_{\mathrm{s}} = 1/T_{\mathrm{S}}$ is the sub-carrier spacing, and $h(\tau,t)$ is the channel impulse response (CIR). In the general case, the demodulated symbol can be written as follows [4]:

$$\hat{d}_{l,i} = \hat{d}^{\mathrm{U}}_{l,i} + \hat{d}^{\mathrm{ICI-CIG}}_{l,i} + \hat{d}^{\mathrm{ICI-CTC}}_{l,i} + \hat{d}^{\mathrm{ISI}}_{l,i}, \quad (2)$$

where $\hat{d}^{\mathrm{U}}_{l,i}$, $\hat{d}^{\mathrm{ICI-CIG}}_{l,i}$, $\hat{d}^{\mathrm{ICI-CTC}}_{l,i}$ and $\hat{d}^{\mathrm{ISI}}_{l,i}$ are the useful symbol, the ICI contribution caused by the insufficient guard length, the ICI contribution caused by the time variations of the channel, and the ISI contribution, respectively. In the following, we consider the system in the different cases of the guard interval length conditions and the channel models.

A. Sufficient guard length and time-varying channels

In this case, the ISI and the ICI-CIG contributions does not occur. The expression of the ICI-CTC power is established by taking the autocorrelation of the ICI-CTC contribution. The final result of the calculation of the ICI-CTC power is obtained by Russel and Stüber [8]:

$$P_{\mathrm{ICI-CTC}} = \frac{E_{\mathrm{S}} \cdot E_{\mathrm{h}}}{T_{\mathrm{S}}^2} \sum_{n=0, n\neq l}^{N_{\mathrm{C}}-1} \int_{t=0}^{T_{\mathrm{S}}} \int_{t'=0}^{T_{\mathrm{S}}} R_t(t-t') \cdot e^{j2\pi(n-l)f_{\mathrm{s}}(t-t')} dt dt', \quad (3)$$

where $R_t(\Delta t), \Delta t = t - t'$, is the time-correlation function of the channel transfer function (CTF), and E_{h} is the normalized channel variance [8] which can be written as:

$$E_{\mathrm{h}} = \int_{\tau=0}^{\tau_{\max}} \rho(\tau) d\tau, \quad (4)$$

where $\rho(\tau)$ is the multi-path channel profile.

B. Insufficient guard length and time-invariant channels

In this case, the ICI-CTC contribution is not present. The analytical results of the ICI-CIG and the ISI powers are presented in [4]. If the difference between the maximal time delay of the channel and the guard interval length is much more smaller than the OFDM symbol duration ($\tau_{\max} - T_{\mathrm{G}} \ll T_{\mathrm{S}}$), then the total interference power is well approximated as follows [4]:

$$P_{\mathrm{I}} = P_{\mathrm{ICI-CIG}} + P_{\mathrm{ICI-CIG}} \approx \frac{2E_{\mathrm{S}}}{T_{\mathrm{S}}} \int_{t=0}^{\tau_{\max}-T_{\mathrm{G}}} \int_{\tau=t+T_{\mathrm{G}}}^{\tau_{\max}} \rho(\tau) d\tau dt, \quad (5)$$

where E_{S} and $\rho(\tau)$ are respectively the transmitted symbol power and the multi-path channel profile.

C. Insufficient guard length and time-varying channels

In the more general case, all the components in Eq. (2) must be taken into account. To analyze the effects of the part of the CIR within the GI and the part outside GI on the demodulated symbol, the CIR is truncated respectively into two parts. The first truncated channel $h_1(\tau, t)$ is the part within the guard interval of the CIR $h(\tau, t)$, and the second truncated channel $h_2(\tau, t)$ is the part outside.[1] According to the locations of the first and the second truncated channel, the integration in the first term of Eq. (1) with respect to the variable τ is devided in two periods. The first period is within $0 \le \tau \le T_{\mathrm{G}}$ and the second period is within $T_{\mathrm{G}} < \tau \le \tau_{\max}$. The second term of Eq. (1) is the ISI contribution and denoted as $\hat{d}_{l,i}^{\mathrm{ISI}}$. Then, Eq. (1) is rewritten as:

$$\begin{aligned}\hat{d}_{l,i} = &\frac{1}{T_{\mathrm{S}}} \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \left\{ \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i} \int_{\tau=0}^{T_{\mathrm{G}}} h_1(\tau, t) g(t-\tau-iT'_{\mathrm{S}}) \right.\\ &\left. \cdot e^{\frac{-j2\pi n\tau}{T_{\mathrm{S}}}} d\tau \right\} e^{\frac{j2\pi(n-l)(t-iT'_{\mathrm{S}})}{T_{\mathrm{S}}}} dt + \frac{1}{T_{\mathrm{S}}} \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}}\\ &\left\{ \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i} \int_{\tau=T_{\mathrm{G}}}^{\tau_{\max}} h_2(\tau, t) g(t-\tau-iT'_{\mathrm{S}}) e^{\frac{-j2\pi n\tau}{T_{\mathrm{S}}}} d\tau \right\}\\ &\cdot e^{\frac{j2\pi(n-l)(t-iT_{\mathrm{S}})}{T_{\mathrm{S}}}} dt + \hat{d}_{l,i}^{\mathrm{ISI}}. \quad (6)\end{aligned}$$

In the first term of Eq. (6), it can be verified that $g(t-\tau-iT'_{\mathrm{S}}) = 1$ for all t and τ in the integration bounds. Thus, the integration result with respect to τ is obviously the CTF of the first truncated channel $H_1(nf_{\mathrm{s}}, t)$ on n-th subcarrier. To analyse the second term of Eq. (6), we separate the integration with respect to t into two intervals. The first integration interval is $iT'_{\mathrm{S}} \le t < iT'_{\mathrm{S}} + \tau_{\max} - T_{\mathrm{G}}$. The second integration interval is $iT'_{\mathrm{S}} + \tau_{\max} - T_{\mathrm{G}} \le t \le iT'_{\mathrm{S}} + T_{\mathrm{S}}$. In the second integration interval, it is straightforward to confirm, that $g(t-\tau-iT'_{\mathrm{S}}) = 1$ for $\forall \tau \in (T_{\mathrm{G}}, \tau_{\max})$. Therefore, equation (6) can be represented as

$$\begin{aligned}\hat{d}_{l,i} = &\frac{1}{T_{\mathrm{S}}} \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \left\{ \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i} H_1(nf_{\mathrm{s}}) \right\} e^{\frac{j2\pi(n-l)(t-iT'_{\mathrm{S}})}{T_{\mathrm{S}}}} dt\\ &+ \frac{1}{T_{\mathrm{S}}} \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}} \left\{ \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i} \int_{\tau=T_{\mathrm{G}}}^{\tau_{\max}} h_2(\tau, t) \right.\\ &\left. \cdot g(t-\tau-iT'_{\mathrm{S}}) e^{\frac{-j2\pi n\tau}{T_{\mathrm{S}}}} d\tau \right\} e^{\frac{j2\pi(n-l)(t-iT'_{\mathrm{S}})}{T_{\mathrm{S}}}} dt\\ &+ \frac{1}{T_{\mathrm{S}}} \int_{t=iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \left\{ \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i} H_2(nf_{\mathrm{s}}, t) \right\}\\ &\cdot e^{\frac{j2\pi(n-l)(t-iT'_{\mathrm{S}})}{T_{\mathrm{S}}}} dt + \hat{d}_{l,i}^{\mathrm{ISI}} \quad (7)\end{aligned}$$

The useful symbol can be picked out from the first three terms of Eq. (7) by setting $n = l$. Then the integration bounds with respect to τ in the second term can be reduced since $g(t-\tau-iT'_{\mathrm{S}})$ is equal to zero in a certain interval and equal to one otherwise. Afterwards, the useful symbol is obtained as follows:

$$\begin{aligned}\hat{d}_{l,i}^{\mathrm{U}} = &\frac{d_{l,i}}{T_{\mathrm{S}}} \left\{ \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} H_1(lf_{\mathrm{s}}, t) dt \right.\\ &+ \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}} \int_{\tau=T_{\mathrm{G}}}^{t-iT'_{\mathrm{S}}+T_{\mathrm{G}}} h_2(\tau, t) e^{-j2\pi lf_{\mathrm{s}}\tau} d\tau dt\\ &\left. + \int_{t=iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} H_2(lf_{\mathrm{s}}, t) dt \right\}. \quad (8)\end{aligned}$$

The autocorrelation of $\hat{d}_{l,i}^{\mathrm{U}}$ is used to calculate the useful power P_{U}. The mathematical expression of the useful power is obtained as follows [see derivation in Appendix]:

$$\begin{aligned}P_{\mathrm{U}} = &\frac{E_{\mathrm{S}}}{T_{\mathrm{S}}^2} \left\{ E_{\mathrm{h}_1} \int_{t=0}^{T_{\mathrm{S}}} \int_{t'=0}^{T_{\mathrm{S}}} R_t(t-t') dt' dt \right.\\ &+ E_{\mathrm{h}_2} \int_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int_{t'=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} R_t(t-t') dt' dt\end{aligned}$$

[1]See Fig. 1 in [4].

$$+ \quad 2 \int\limits_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int\limits_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau=T_{\mathrm{G}}}^{t'+T_{\mathrm{G}}} \rho(\tau) R_t(t-t') d\tau dt' dt$$

$$+ \int\limits_{t=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau=T_{\mathrm{G}}}^{\min\{\mathrm{t}+\mathrm{T_G},\mathrm{t}'+\mathrm{T_G}\}} \rho(\tau)$$

$$\cdot \quad R_t(t-t') d\tau dt' dt \Big\}, \tag{9}$$

where T_{S}, T_{G}, $\tau_{\max}$, and E_{S} are respectively the OFDM symbol duration, the guard interval length, the maximal time delay of the channel and the transmitted symbol energy. The denotation $E_{\mathrm{h}_1} = \int_{\tau=0}^{T_{\mathrm{G}}} \rho(\tau) d\tau$ and $E_{\mathrm{h}_2} = \int_{\tau=T_{\mathrm{G}}}^{\tau_{\max}} \rho(\tau) d\tau$ are defined as the channel variance of the first and the second truncated channel ($E_{\mathrm{h}} = E_{\mathrm{h}_1} + E_{\mathrm{h}_2}$). The total power P_{T} of the signal at the output of the FFT consists of the useful power P_{U} and the interference power P_{I}, and it is always equal to the product of the transmitted symbol energy E_{S} and the channel variance E_{h}

$$P_{\mathrm{T}} = E_{\mathrm{h}} \cdot E_{\mathrm{S}}. \tag{10}$$

Therefore, we can calculate the interference power from the useful power by using the following relationship:

$$P_{\mathrm{I}} = P_{\mathrm{T}} - P_{\mathrm{U}}. \tag{11}$$

In the next section, we show how to obtain the SER of an OFDM system using 16-QAM on each sub-carrier.

III. Symbol error ratio of uncoded OFDM systems

The symbol error ratio of OFDM systems depends on many factors. For simplification, the uncoded system is taken into consideration. 16-QAM modulation on each sub-carrier is used. The SER for M-ary QAM on AWGN channel is given in [7] as follows:

$$\begin{aligned} P(\gamma_s) &= 2\frac{\sqrt{M}-1}{\sqrt{M}} \mathrm{erfc}\left(\sqrt{\frac{3}{2(\mathrm{M}-1)}\gamma_{\mathrm{s}}}\right) \\ &- \left(\frac{\sqrt{M}-1}{\sqrt{M}} \mathrm{erfc}\left(\sqrt{\frac{3}{2(\mathrm{M}-1)}}\gamma_{\mathrm{s}}\right)\right)^2, \end{aligned} \tag{12}$$

where γ_s is the instantaneous signal-to-noise ratio (SNR) per symbol and erfc($\cdot$) is the complementary error function. For 16-QAM, M is replaced by 16 in equation (12). To obtain the error probabilities on the time-varying channel with Rayleigh fading, the SER must be averaged over the probability density function of γ_s given as follows [6]

$$p(\gamma_s) = \frac{1}{\bar{\gamma_s}} \mathrm{e}^{-\gamma_{\mathrm{s}}/\bar{\gamma_{\mathrm{s}}}} \qquad \gamma_{\mathrm{s}} \geq 0, \tag{13}$$

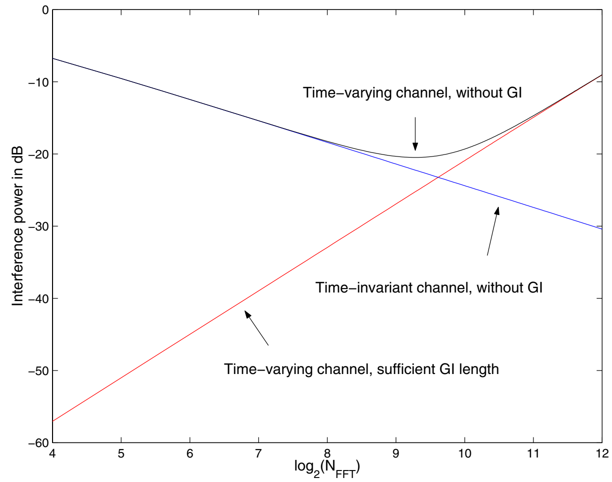

Fig. 1. Interference power versus FFT length for different cases of guard interval length conditions and channel model.

where $\bar{\gamma_s}$ is the average SNR. In the OFDM sytem, the use of the guard interval leads to different lengths of the impulse responses of transmitting and receiving filters, and results in *mismatched filtering* [3]. Consequently, the average SNR is lost by a factor of $T_{\mathrm{S}}/T'_{\mathrm{S}}$. In general case, the average SNR of an OFDM system is

$$\mathrm{SNR} = \frac{\mathrm{P_U}}{\mathrm{P_T} - \mathrm{P_U} + \mathrm{N_0}} \cdot \frac{\mathrm{T_S}}{\mathrm{T'_S}}, \tag{14}$$

where N_0 is the noise variance. In the free additive noise condition, the system suffers only from the interference distortion. The signal-to-interference ratio (SIR) for an OFDM system without GI is

$$\mathrm{SIR} = \frac{\mathrm{P_U}}{\mathrm{P_T} - \mathrm{P_U}}. \tag{15}$$

Replacing $\bar{\gamma_s}$ in Eq. (13) by SIR in (15), then evaluating the following integral

$$\mathrm{SER} = \int\limits_0^{\infty} \mathrm{P}(\gamma_{\mathrm{s}}) \mathrm{p}(\gamma_{\mathrm{s}}) \mathrm{d}\gamma_{\mathrm{s}}. \tag{16}$$

The integration result of Eq. (16) gives the SER due to interference distortion. So, Eq. (16) provides an estimation to the SER due to interference distortion.

IV. Simulation results

In this section, all the simulations were performed within the HiperLAN2 framework [1] and using 16-QAM constellations. To analyse the dependence of the interference powers on the OFDM symbol duration, the FFT length is varried. The channel model is adopted from the channel model A described in [2]. The maximal Doppler frequency on each tap for the case of time-varying channel is selected to be 1000 Hz with the purpose that the FFT

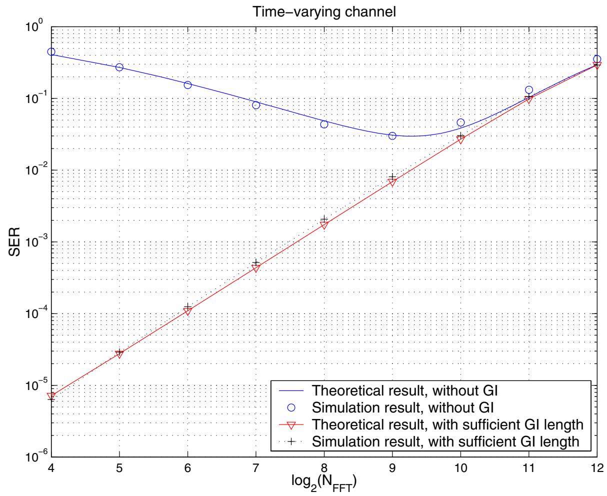

Fig. 2. The SER caused by ISI and ICI interferences for 16-QAM on a time-varying channel; Additive noise is not taken into account.

length does not require to be chosen too large to see the effect of the time variations of the channel. Otherwise, the computation time would be rather long.

Fig. 1 shows the theoretical interference power for different cases of GI conditions and the channel model.

The simulation results of SER are plotted in Fig. 2, where the theoretical results are also given for reference. It can be seen that the analytical results of the SER in Fig. 2 reflect also the analytical results of interference powers in Fig. 1, where the minimum point of interference power corresponds to the minimum point of SER. It can be concluded that the interference power can be caused by different effects which do not relate together (insufficient GI length or time variations of the channel), but their influences on the SER of the system are identical.

V. Conclusion

The paper provides a system analysis concerning interference power and symbol error ratio for an OFDM system in the case of insufficient guard interval length and on a time-varying channel. Even though the interference contributions are caused from different sources, their influences on the SER of the system are equal. In the case of insufficient guard interval length and time-varying channel, the system has optimal performance, when the two different interference contributions are equal. This point is important for the system design consideration.

Appendix

The expectation of the square value of the useful symbol in Eq. (8) expresses the average useful power as follows:

$$P_U = E\left[\left(\hat{d}^U_{l,i}\right)^* \cdot \hat{d}^U_{l,i}\right]. \tag{17}$$

The useful symbol in Eq. (8) comprises three terms. The first term is derived from the first truncated channel, and the two last terms are generated from the second truncated channel. Considering both the cross-correlation of the two truncated channels vanishes and the expression of the autocorrelation of the transmitted data symbol

$$E\left[d^*_{l,i}\, d_{l',i'}\right] = \begin{cases} E_{\mathrm{S}} & : \text{ for } \quad \text{l=l', i=i'} \\ 0 & : \text{ otherwise,} \end{cases} \tag{18}$$

equation (17) can be expanded as follows:

$$\begin{aligned} P_U \;=\; & \frac{E_{\mathrm{S}}}{T_{\mathrm{S}}^2}\Bigg\{ \int\limits_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \int\limits_{t'=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \\ & E\left[H^{(1)^*}(lf_{\mathrm{s}},t)\, H^{(1)}(lf_{\mathrm{s}},t')\right] dt'\, dt \\ + & \int\limits_{t=iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \int\limits_{t'=iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \\ & E\left[H^{(2)^*}(lf_{\mathrm{s}},t) H^{(2)}(lf_{\mathrm{s}},t')\right] dt'\, dt \\ + & \; 2 \int\limits_{t=iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \int\limits_{t'=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau'=T_{\mathrm{G}}}^{t'-iT'_{\mathrm{S}}+T_{\mathrm{G}}} \\ & E\left[H^{(2)^*}(lf_{\mathrm{s}},t)\, h^{(2)}(\tau',t')\right] e^{-j2\pi l f_{\mathrm{s}}\tau'} d\tau'\, dt'\, dt \\ + & \int\limits_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}} \int\limits_{t'=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau=T_{\mathrm{G}}}^{t-iT'_{\mathrm{S}}+T_{\mathrm{G}}} \int\limits_{\tau'=T_{\mathrm{G}}}^{t'-iT'_{\mathrm{S}}+T_{\mathrm{G}}} \\ & E\left[h^{(2)^*}(\tau,t)\, h^{(2)}(\tau',t')\right] \\ \cdot \; & e^{j2\pi l f_{\mathrm{s}}(\tau-\tau')} d\tau'\, d\tau\, dt'\, dt \Bigg\} \end{aligned} \tag{19}$$

In the above equation, the presence of the OFDM symbol index i can be omitted by changing the integration bounds. The first and the second terms in Eq. (19) can be further derived as follows:

$$\begin{aligned} P_{U_1} &= \int\limits_{t=0}^{T_{\mathrm{S}}} \int\limits_{t'=0}^{T_{\mathrm{S}}} E\left[H^{(1)^*}(lf_{\mathrm{s}},t)\, H^{(1)}(lf_{\mathrm{s}},t')\right] dt'\, dt \\ &= E_{\mathrm{h}_1} \int\limits_{t=0}^{T_{\mathrm{S}}} \int\limits_{t'=0}^{T_{\mathrm{S}}} R_t(t-t')\, dt'\, dt, \end{aligned} \tag{20}$$

and

$$\begin{aligned} P_{U_2} = & \int\limits_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int\limits_{t'=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} E\Big[H^{(2)^*}(lf_{\mathrm{s}},t) \\ & \cdot H^{(2)}(lf_{\mathrm{s}},t')\Big]\, dt'\, dt \end{aligned}$$

$$= E_{\mathrm{h}_2} \int\limits_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int\limits_{t'=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} R_t(t-t')\, dt'\, dt. \qquad (21)$$

After replacing $H^{(2)^*}(lf_{\mathrm{s}}, t)$ by

$$H^{(2)}(lf_{\mathrm{s}}, t) = \int\limits_{\tau=T_{\mathrm{G}}}^{\tau_{\max}} h^{(2)}(\tau, t) e^{-j2\pi l f_{\mathrm{s}} \tau} d\tau \qquad (22)$$

in the third term of (19), this term can be written as follows:

$$\begin{aligned} P_{U_3} &= \int\limits_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int\limits_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau'=T_{\mathrm{G}}}^{t'+T_{\mathrm{G}}} E\Big[H^{(2)^*}(lf_{\mathrm{s}}, t) \\ &\quad \cdot h^{(2)}(\tau', t')\Big] e^{-j2\pi l f_{\mathrm{s}} \tau'} d\tau'\, dt'\, dt \\ &= \int\limits_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int\limits_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau=T_{\mathrm{G}}}^{\tau_{\max}} \int\limits_{\tau'=T_{\mathrm{G}}}^{t'+T_{\mathrm{G}}} E\Big[h^{(2)^*}(\tau, t) \\ &\quad \cdot h^{(2)}(\tau', t')\Big] e^{-j2\pi l f_{\mathrm{s}}(\tau-\tau')} d\tau'\, d\tau\, dt'\, dt. \qquad (23) \end{aligned}$$

With the definition of the autocorrelation function of the CIR [6]:

$$E\Big[h^*(\tau, t)\, h(\tau+\Delta\tau, t+\Delta t)\Big] = r(\tau, \Delta t)\, \delta(\Delta\tau), \qquad (24)$$

it follows

$$\begin{aligned} P_{U_3} &= \int\limits_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int\limits_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau=T_{\mathrm{G}}}^{\tau_{\max}} \int\limits_{\tau'=T_{\mathrm{G}}}^{t'+T_{\mathrm{G}}} r(\tau, t-t') \\ &\quad \cdot \delta(\tau-\tau') e^{-j2\pi l f_{\mathrm{s}}(\tau-\tau')} d\tau'\, d\tau\, dt'\, dt \\ &= \int\limits_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int\limits_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau=T_{\mathrm{G}}}^{t'+T_{\mathrm{G}}} r(\tau, t-t')\, d\tau\, dt'\, dt. \end{aligned} \qquad (25)$$

According to [6], the autocorrelation function of $H(f, t)$, $R(\Delta f, t-t')$, is the Fourier transformation of the autocorrelation function of $h(\tau, t)$, $r(\tau, t-t')$, with respect to τ. It can be proven that $R(\Delta f, t-t')$ can be computed by the product of two function $R_f(\Delta f)$ and $R_t(t-t)$, therefore $r(\tau, t-t')$ can be written as

$$r(\tau, t-t') = \rho(\tau)\, R_t(t-t'). \qquad (26)$$

Replacing the expression of the autocorrelation function of the CIR from Eq. (24) into the last term of Eq. (19), this term becomes

$$P_{U_4} = \int\limits_{t=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau=T_{\mathrm{G}}}^{\min\{t+T_{\mathrm{G}}, t'+T_{\mathrm{G}}\}} r(\tau, t-t')\, d\tau\, dt'\, dt. \qquad (27)$$

Substituting the results from Eq. (20), (21), (25), and (27) in Eq. (19), and using the expression of the autocorrelation of the channel impulse response in Eq. (26), yields the mathematical expression of the useful power provided in Eq. (9).

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