Estimation of the Channel-Impulse-Response Length for Adaptive OFDM Systems Based on Information Theoretic Criteria

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Abstract—**This paper proposes a new scheme based on information theoretical criteria to estimate the Channel-Impulse-Response (CIR) length, whose accurate estimate represents one of the most important tasks for the realization of OFDM systems with adaptive guard interval. The difference between the statistical characteristics of the additive noise and the mobile radio channel is exploited for this task. In the proposed algorithm, the calculation of the autocorrelation matrix and their eigenvalues are not required. Therefore, the complexity of the proposed method is low. A modified penalty function is introduced to make the criteria more stable. Performance of the proposed method is evaluated for HiperLAN/2 channels. The simulation results show that the proposed algorithm is robust and delivers a precise estimate for the CIR length even at sever noisy environment with as low signal-to-noise ratio (SNR) as 0 dB. It delivers therefore reliable information for adaptive Guard Interval (GI) based OFDM systems and channel estimation.**

Keywords- **CIR length estimation, information theoretical criteria, OFDM systems.**

I. INTRODUCTION

In OFDM systems, the problem of Inter-Symbol Interference (ISI) is significantly reduced by splitting the high rate symbol stream into several low rate streams (windows), in which a parallel data transmission through multiple orthogonal carriers takes place. A cyclic extension (cyclic prefix) is inserted to the output sequence between adjacent windows. This cyclic extension is known as the GI and its length T_G must be equal to or greeter than the maximum delay excess of the channel. Consequentially, the GI depends on the wireless channel environment, and needs to be selected for the channel worst-case conditions. However using unnecessarily long GI leads to a reduction of the spectral efficiency of the system. In order to achieve the highest possible spectral efficiency, the guard interval should be adapted according to the channel conditions. Such an adaptation process requires a realistic estimation of the CIR length of the wireless channel.

Additive noise usually cause an overestimation of the CIR length. The difference between the statistical characteristics of the additive noise and that of the multipath channel is that the CIR coefficients are only located in the window of the CIR length whereas the additive noise per channel tap is uniformly distributed over the whole length of the estimated CIR.

Different techniques for estimating the CIR length are available in the literatures. A method has been recently introduced in [1], in which the CIR length is estimated based on the difference between the statistical characteristics of the radio channel and that of the additive noise. It needs however a long averaging duration. A joint estimation of the symbol timing and the channel length has been introduced in [2]. The method is based on the maximum likelihood principle and generalized Akaike information criteria of eigenvalues, It shows that the CIR length is usually underestimated. In [3] a cyclic-prefix based delay-spread estimation technique for wireless OFDM systems has been proposed. A Channel estimation method based on parametric channel modeling has been proposed in [4], which is based on the Minimum Descriptive Length (MDL) of eigenvalues of the averaged autocorrelation matrix.

This paper proposes a novel CIR-length estimation technique. The ability of information theoretic criteria for separating the statistical characteristics of the additive noise space from that of the mobile radio channel space is used to estimate the CIR length. Precise results are obtained using a modified penalty function. These criteria are applied directly to the estimated Delay Power Spectrum (DPS) without the need to determine the eigenvalues of the autocorrelation matrix, which reduces the computation complexity of the algorithm. By minimizing the MDL value, the optimal value of the channel length can be consistently and efficiently determined.

The paper is organized as follows: Section II explains the OFDM system model. The principles of the proposed CIR-length estimation algorithm are presented in Section III. The performance of the CIR estimator is shown in Section IV. Finally, concluding remarks are given in Section V.

II. OFDM SYSTEM MODEL

The channel estimation in OFDM systems can be performed by using pilot symbols. These pilots are inserted into data streams at the transmitter, and removed afterwards at the receiver. The investigated system is depicted in Fig. 1. Assuming that the channel is time invariant over one OFDM symbol, the estimated Channel Transfer Function (CTF) $H'_{p,i}$ at the position of the pilot symbol $S_{p,i}$ is obtained by dividing the received pilot symbol $(R_{p,i}+w_{p,i})$ by the known pilot symbol $S_{p,i}$

$$H'_{p,i} = \frac{R_{p,i}}{S_{p,i}} + \frac{w_{p,i}}{S_{p,i}} \quad (1)$$

where p and i indicate the p^{th} subcarrier and the i^{th} OFDM symbol, respectively, and $w_{p,i}$ denotes the additive white Gaussien noise (AWGN).

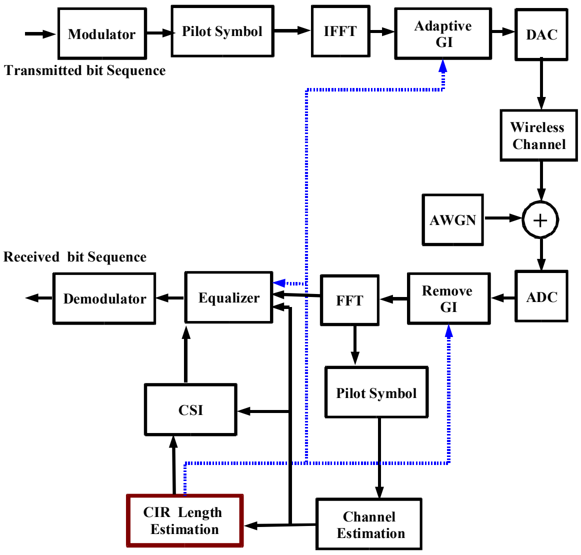

Fig. 1 The OFDM system

An average estimate for the CTF is obtained by time-averaging the first estimated CTF over I OFDM symbols [4]. The channel coefficients are assumed to be constant over the averaging range:

$$\hat{H}_p = \frac{1}{I}\sum_{i=1}^{I} H'_{p,i} \quad (2)$$

The averaging procedure is repeated M times to reduce the effect of noise. The associated estimated CTF at the positions of pilot symbols per one OFDM symbol can be represented in vector form as

$$\vec{\hat{H}}_i = \left[\hat{H}_{0,i}\hat{H}_{1,i},\ldots,\hat{H}_{N_p-1,i}\right]^T \quad (3)$$

where N_p is the number of pilot symbols per one OFDM symbol and T denotes the transpose operation. The matrix formulation of the CTF could be written as [4]

$$\vec{\hat{H}}_i = \mathbf{F}\vec{\hat{h}}_i + \vec{W}_i \quad (4)$$

where $\mathbf{F}$ is an $(N_p \times N_p)$ discrete-Fourier-transform (DFT) matrix . It is given as

$$\mathbf{F} = \begin{bmatrix} F_{0,0} & F_{0,1} & \cdots & F_{0,N_p-1} \\ F_{1,0} & F_{1,1} & \cdots & F_{1,N_p-1} \\ \vdots & \vdots & \ddots & \vdots \\ F_{N_p-1,0} & F_{N_p-1,0} & \cdots & F_{N_p-1,N_p-1} \end{bmatrix} \quad (5)$$

The element (n,k) of $\mathbf{F}$ reads

$$F_{n,k} = e^{j2\pi nk/N_p} \quad (6)$$

Where $n,k \in \{0,1,\ldots.N_p\text{-}1\}$. The vector $\vec{\hat{h}}_i$ which represents N_p samples of the CIR at the i^{th} OFDM symbol is the counterpart vector of $\vec{\hat{H}}_i$ in the time domain. It is given by

$$\vec{\hat{h}}_i = \left[\hat{h}_{0,i},\hat{h}_{1,i},\ldots,\hat{h}_{N_p-1,i}\right]^T \quad (7)$$

The distance between the channel tap indices k and k+1 is assumed to be equal to the sampling interval T_a of the system.

Although the above procedure results in an estimation for the CIR, the related CIR length may be overestimated due to the additive noise, which produces some additional taps beyond the noise-free CIR length. As an example, Fig. 2 shows a typical estimate for a noisy CIR (SNR = 0dB), in which the estimated CIR length is 15 sampling intervals. The noise-free CIR length for this case is 8 sampling intervals only.

In general, the noise-free CIR length L is less than or equal the estimated (noisy) one N_p. In order to reduce the noise effect, let us define a new CIR estimate according to:

$$\hat{h}^L_{k,i} = \begin{cases} \hat{h}_{k,i} & 0 \le k < L \\ 0 & L \le k < N_p \end{cases} \quad (8)$$

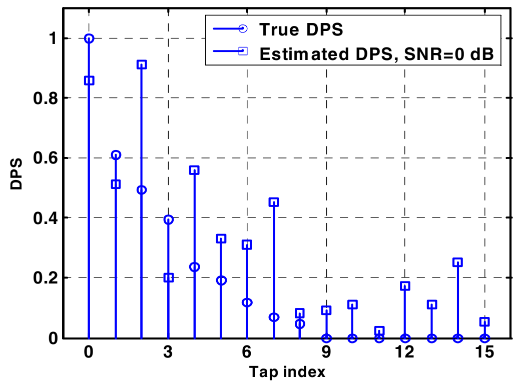

Fig. 2 Estimated DPS distorted by additive noise of SNR =0 dB

III. NEW CIR-LENGTH ESTIMATION ALGORITHM

Since the channel is assumed to be represented by an uncorrelated scattering process, the autocorrelation matrix $\mathbf{R}_{HH}$ of the CTF can be written as

$$\mathbf{R}_{HH} = \mathbf{E}\left[\vec{H}.\vec{H}^{\mathbf{H}}\right] \tag{9}$$

A Singular Value Decomposition (SVD) of $\mathbf{R}_{HH}$ [5] results in

$$\mathbf{R}_{HH} = \mathbf{U}^{\mathbf{H}}\mathbf{\Lambda}\mathbf{U} \tag{10}$$

where $\mathbf{H}$ denotes the Hermitian transpose, and $\mathbf{\Lambda}$ is $(N_\mathrm{p} \times N_\mathrm{p})$ diagonal matrix with elements

$$P_k = E\left[\hat{h}_k.\hat{h}_k^{\,*}\right] + \sigma^2 \qquad k \in \left[0,...N_\mathrm{p}-1\right] \tag{11}$$

that represent the singular values of $\mathbf{R}_{HH}$. The parameter σ^2 represents the noise variance. The elements of $\mathbf{\Lambda}$ are then N_p samples of the estimated noisy DPS $P(\tau)$. The N_p-dimentional space to which $\mathbf{R}_{HH}$ belongs consists generally of two orthogonal spaces: A signal space and a noise space. The CIR length L represents in fact the dimensionality of the signal space. The last (smallest) $(N_\mathrm{p} - L)$ elements of the diagonal elements of $\mathbf{\Lambda}$ characterize the noise space and should therefore be of the same order of magnitude as σ^2. The matrix $\mathbf{\Lambda}$ can be partitioned as

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}_{CIR} & 0 \\ 0 & \mathbf{\Lambda}_N \end{bmatrix} \tag{12}$$

where $\mathbf{\Lambda}_{CIR}$ is $L\times L$ diagonal matrix containing the L elements P_k=E$[h_k.h_k^*]$ $+\sigma_\kappa^2$, $k\in[0,\ldots, L-1]$, and $\mathbf{\Lambda}_N$ is $(N_\mathrm{p} - L)\times(N_\mathrm{p} - L)$ diagonal matrix containing the $(N_\mathrm{p} - L)$ elements σ_κ^2 , $k\in[L,\ldots, N_\mathrm{p}-1]$. The latter characterizes the additive noise. Since the additive noise is white, the noise terms σ_κ^2 are approximately equal, i.e.,

$$\sigma_0^2 \approx \ldots \approx \sigma_{L-1}^2 \approx \sigma_L^2 \approx \ldots \approx \sigma_{N_\mathrm{p}-1}^2 \,. \tag{13}$$

Let us consider now the structural and statistical properties of $\mathbf{R}_{HH}$. The conditional probability density function of the statistically independent complex Gaussian random vector $\vec{H}$ is given by [6]

$$f(\vec{H}|\phi) = \frac{1}{\pi^{N_\mathrm{p}}\, det(\mathbf{R}_{HH})}\, \mathrm{e}^{\left\{-\vec{H}^H[\mathbf{R}_{HH}]^{-1}\vec{H}\right\}} \tag{14}$$

where $\phi = [P_1, \ldots, P_k, \sigma^2, \mathbf{f}_1^T, \ldots, \mathbf{f}_k^T]^T$ is the parameter vector of the model, $\mathbf{f}_1$,., $\mathbf{f}_k$ are the columns of the $\mathbf{F}$-matrix, and *det* denotes the determinant of the matrix. The corresponding log-likelihood function is given by

$$L(\phi) = -N_\mathrm{p}\log(det(\mathbf{R}_{HH})) - tr\left[\mathbf{R}_{HH}\right]^{-1}\mathbf{R}_{HH} \tag{15}$$

where *tr* represent the trace of the matrix**.** The close expression of the maximum log-likelihood function (ML) in (15) has been obtained in [7], and is given by

$$L(\phi^{(k)}) = -(N_\mathrm{p} - k)M_B \log\left(\frac{G(P_{k+1},\ldots,P_{N_\mathrm{p}})}{A(P_{k+1},\ldots,P_{N_\mathrm{p}})}\right) \tag{16}$$

where the functions G and A are the geometric and arithmetic mean of their arguments, respectively [8]. The value of M_B needs to be selected to provide a balance between resolution and stability of the criteria. The results of computer simulations have shown that optimal values for M_B lie between $I/2$ and $I/3$. Equation (16) shows that there is no need to find the autocorrelation matrix or its eigenvalues. This significantly reduces the computation complexity of the proposed algorithm.

Utilizing the ML-function for estimating L results in its maximum allowable value. On the other hand, utilizing information theoretic criteria for the model selection, includes an additional bias correction term (penalty function). It is added to the log-likelihood function in order to bias the over model estimation. This results in the MDL-function, which is given in [7] as

$$\mathrm{MDL}(k) = L(\phi) + f(k, N_\mathrm{p}) \tag{17}$$

where the penalty function $f(k, N_\mathrm{p})$ is given by

$$f(k, N_\mathrm{p}) = \frac{1}{2}k(2N_\mathrm{p} - k)\log(M_B) \tag{18}$$

The CIR length L is now taken to be the value of $k\in\{0,1,\ldots N_\mathrm{p}-1\}$ for which MDL(k) is minimum.

It is worth noting that using the above penalty function results in a higher slope of the bias correction at low SNR. Therefore, utilizing the MDL-function underestimates the CIR length. In order to reduce the slop of the penalty function (and hence that of the MDL one) the following modified penalty function can be used to bias the minimum of the MDL to a better value of the CIR length:

$$f(k, N_\mathrm{p}) = \frac{1}{4}k(2N_\mathrm{p} - k)\log(M_B) + k \tag{19}$$

The simulation results show that this modified penalty function provides much more precise results.

III. SIMULATION RESULTS AND DISCUSION

The simulated OFDM system parameters have been selected similar to that of HiperLAN/2 [9,10]. These read:

- Bandwidth of the system $B = 20$ MHz,
- Sampling interval: $T_a = 1/B = 50$ ns,
- FFT length: N=64,
- OFDM symbol duration: $T_s = N.T_a = 3.2$ µs.

The simulated channel is a typical indoor one (channel model A in [9]). The coefficients of the discrete multipath are given in Table 1.

Table 1 Characteristics of the modified HIPERLAN/2 channel [9, 10]

Path index k	Time delay τ_k (nsec)	Path Power (Normalized)
1	0	1
2	50	0.6095
3	100	0.4945
4	150	0.3940
5	200	0.2371
6	250	0.1900
7	300	0.1159
8	350	0.0699
9	400	0.0462

The performance of MDL depends on the selection of the two parameters M and I. The number of iterations M is first kept constant (M = 5), while the averaging length I is varied. The proposed algorithm has been tested for the OFDM system described above in the presence of strong noise (SNR = 5 and 0 dB). Fig. 3 shows that an averaging length over 300 is required to estimate the CIR length accurately for SNR = 0 dB. For SNR = 5 dB a minimum averaging length of only 200 is required.

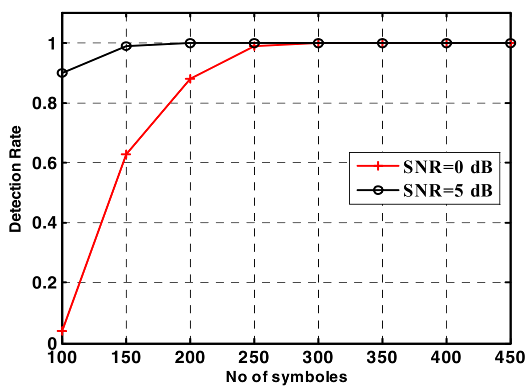

Fig. 3 The CIR length detection rate versus averaging length

Fig.4 Shows the results of varying the number of averaging process M for a fixing SNR of 0 dB. At I = 350, the CIR length is very accurately estimated at M = 5 while at I= 250 a value of M = 7 is needed.

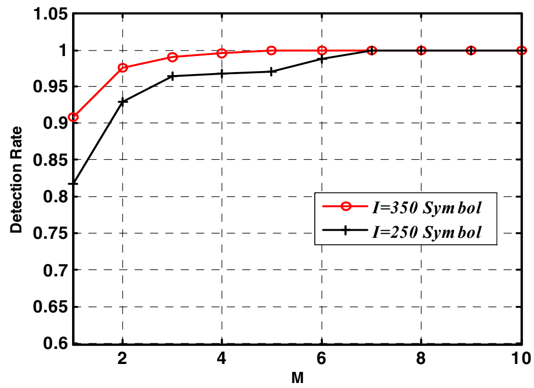

Fig. 4 The CIR length detection rate versus number of averaging

The performance of the algorithm with respect to the SNR is presented in Fig. 5. It shows the detection rate using 500 independent simulations for SNR ranging between –5 and 20dB. The lower the SNR value, the longer the averaging length which is needed to accurately detect the CIR length.

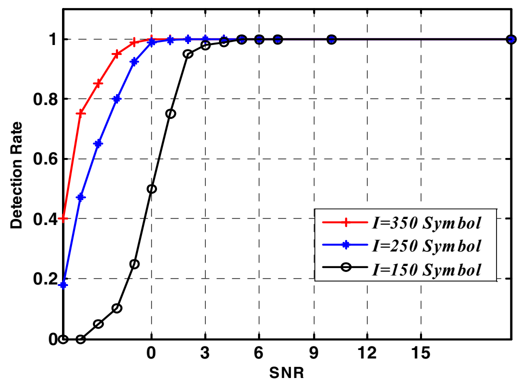

Fig. 5 The CIR length detection rate versus SNR

Fig. 6 shows the MDL functional dependence on the number of channel taps (L) for different simulations. These results justify that the MDL-function reaches its minimum at the noise-free CIR length.

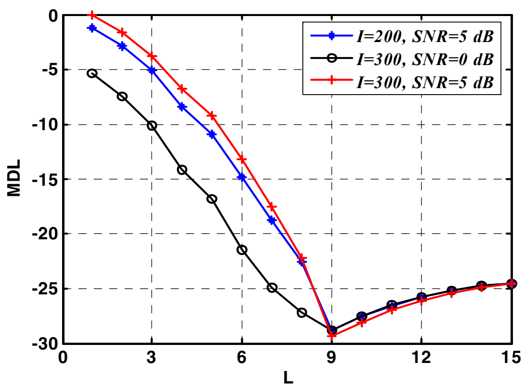

Fig. 6 Simulation results of MDL in different cases of averaging length and SNR

The difference between the performance of the two penalty functions for predicting the 9-taps CIR is shown in Fig. 7 where (18) leads to an underestimated value of the CIR length. Using (19) on the other hand results in an MDL-minimum which coincides with the noise-free CIR length. Furthermore, the detection rate for the penalty function in (18) is 99% while that corresponding to (19) is 100%. The stability of the MDL criteria for different simulations is shown in Fig.8 where the minimum takes place at the noise-free length L=9.

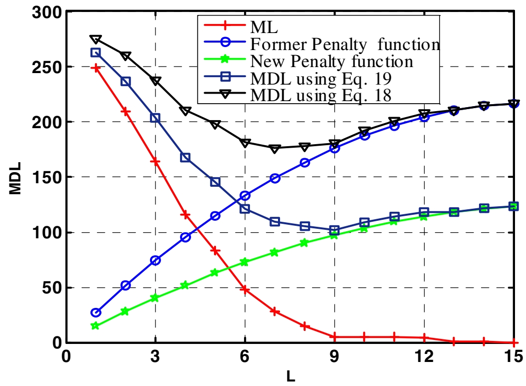

Fig. 7 MDL-function obtained using the original and modified penalty functions

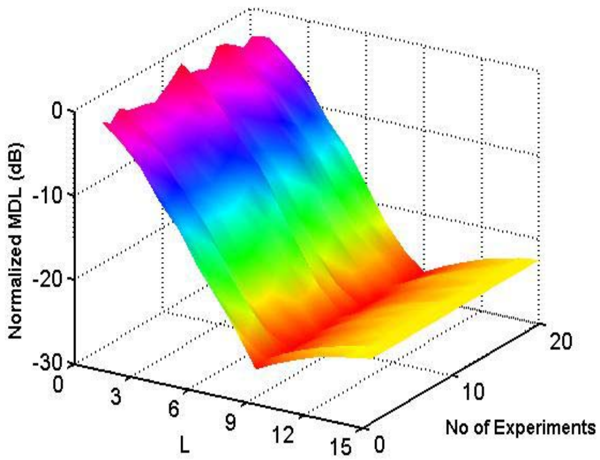

Fig. 8 Stability of the estimation criteria

V. CONCLUSIONS

This paper describes a new method estimating of the CIR length for adaptive OFDM communication systems. It is based on information theoretical criteria for separating the signal and noise spaces. The method is robust against strong additive noise and computationally very efficient, due to the fact that neither the correlation matrix nor its singular values have to be explicitly computed. Simulation results for a typical OFDM system with very low SNR have shown that a robust estimation of the correct CIR length can be obtained.

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