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Practical considerations of optimal three-dimensional indoor localization

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Abstract— **The problem of locating objects and people in indoor environments is lately gaining high interest in research and industry. In this context, the geometrical constellation of source and sensors plays an important role and heavily influences the accuracy for the position estimate. This paper uses existing optimal solutions of the problem of three-dimensional TDOA localization, and relates them to practical application. The extension is necessary, since the optimal solution is only valid for one single position and not for a complete area, where a localization capability is desired. Therefore, it is shown by simulations that the optimum is very useful for localization purposes in typical indoor scenarios. In addition, real-world measurements using an Ultra-Wideband (UWB) localization system have been carried out in two geometrical scenarios. It can be shown that the simulations, which are based on theoretical derivations, are a good approximation of the reality. Finally, it can be demonstrated that the optimal solution can be adapted to typical indoor environments and still yield accurate three-dimensional positioning.**

I. Introduction

Applications, which use the position information of objects or people, are becoming more and more attractive. This is mainly due to the increasing use of mobile devices that are equipped with communication capabilities. Different methods can be applied to solve the localization problem. One possibility is to estimate the position of a radiating source via the analysis of signals received at a certain number of sensors. This approach is normally based on measuring range differences between source and sensors, and subsequently processing the range information into a position estimate via Time Difference Of Arrival (TDOA) algorithms. In this case, the localization accuracy mainly depends on two parameters: the error of the ranging differences and the geometrical arrangement of the sources and sensors. For example, to achieve decimeter accuracy of position estimation it is necessary to use a radio technology, which can achieve decimeter accuracy in ranging. However, the impact of the geometry can be quite severe such that the goal is not reached. Therefore, in this paper the latter parameter is of interest, and previously proposed optimal geometries of sensors are further investigated.

The problem of evaluating the localization accuracy of a given constellation has been studied for a long time. Especially for passive source localization, but also for positioning via satellites and lately for cellular networks, analysis has been carried out based on covariance study of localization errors and estimation theory [1-6]. In addition, research has been performed with the task of optimally placing the sensors for different scenarios, i.e. Direction Of Arrival (DOA) [7-9] or two-dimensional TDOA [9, 10]. Most work is based on minimizing some kind of error variance of the estimation function. Only lately the three-dimensional TDOA localization problem has been addressed [11-13] and an optimal solution as been analytically derived.

Our paper uses the theoretical optimum and verifies its usefulness for practical applications. The extension is necessary, since the optimal solution is only valid for one single position and not for a complete area, where a localization capability is desired. Therefore, it is shown by simulations that the optimum is very useful for localization purposes in typical indoor scenarios. In addition, real-world measurements using an Ultra-Wideband (UWB) localization system have been carried out in two geometrical scenarios. It can be shown that the simulations, which are based on theoretical derivations, are a good approximation of the reality. Further, it can be demonstrated, that careful adaptation of the optimal geometrical solution to the installation conditions of an indoor scenario, will yield accurate position estimation.

The paper is organized as follows: The signal setup and the optimal geometry will be introduced in section II. In section III, the solution is studied under practical considerations by means of simulation. Section IV describes the UWB measurement setup and analyses the different geometrical scenarios in which a source is located. Finally, chapter V derives the mentioned metric and discussed the comparison between the simulations and the measurements.

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II. SETUP AND OPTIMAL GEOMETRY

In a Cartesian coordinate frame the Euclidian distance d_i between mobile source and sensor i is given by

$$d_i = \|\mathbf{x} - \mathbf{x_i}\|, 1 \le i \le N \quad (1)$$

with source position $x = [x,y,z]^T$, i^{th} sensor position $\mathbf{x_i}$, number of sensors N, and $\| \ldots \|$ denoting the L^2 norm. The received signal $s_i(t)$ at sensor i can be modeled as an attenuated and delayed version of the radiated signal $r(t)$ with additive noise:

$$s_i(t) = m_i(d_i) \cdot r(t - t_i) + n_i(t),\ 1 \le i \le N\,, \quad (2)$$

where $m_i(d_i)$ denotes a distance-dependent attenuation factor and t_i the absolute time of arrival at sensor i. The radiated signal $r(t)$ is assumed to be a stationary Gaussian process, the noises $n_i(t)$ are assumed to be identically distributed zero-mean stationary Gaussian processes, and all are mutually uncorrelated.

Time differences Δt_{ij} are measured via e.g. cross-correlation of the received signals at sensor pair (i,j), and are converted to range differences Δd_{ij} by multiplying by the speed of light c.

$$\Delta d_{ij} = c \cdot \Delta t_{ij} = c(t_i - t_j) = d_i - d_j\,, \quad (3)$$

where t_i and t_j are the absolute times of arrival at sensor i and j, respectively. Substituting the distances d_i of (1) into (3) yields $D = N(N-1)/2$ hyperboloid pseudo-range equations:

$$c \cdot \Delta t_{ij} = \|\mathbf{x} - \mathbf{x_i}\| - \|\mathbf{x} - \mathbf{x_j}\|,\ 1 \le i < j \le N\,. \quad (4)$$

From the non-linear system of D (usually over-determined) equations, the general problem is to solve for the mobile unit's position $\mathbf{x}$, given the base stations' positions $\mathbf{x_i}$ and the TDOA measurements Δt_{ij}. The intersection of these hyperbolas corresponds to the position of the mobile unit, if the TDOA measurements Δt_{ij} are exact. Therefore, one way to find a position estimate from TDOA measurements is the direct solution of the set of pseudo-range equations [14-16].

The performance of such estimators is usually evaluated with the lower bound for the covariance matrix of unbiased estimators, the Cramer-Rao Lower Bound (CRLB) [17, pp.30]:

$$Var\{\hat{\mathbf{x}}\} \ge \mathbf{J}_{\hat{\mathbf{x}}}^{-1}\,,$$

where $\mathbf{J}_{\hat{\mathbf{x}}}$ is the Fisher information matrix. The CRLB has been derived for the case of position estimates via both, TDOA and Time-Of-Arrival (TOA) measurements [8, 9, 16, 17, p.45, 18]. Note, that no difference exists between the TOA and TDOA case [19].

In the navigation field, a common expression for analyzing the geometrical constellation is the non-dimensional quantity set Position Dilution Of Precision ($PDOP$) [4, pp.262, 5, pp.36]. It describes how measurement errors translate into position errors:

$$PDOP = \frac{RMS\ position\ error}{RMS\ measurement\ error}\,, \quad (5)$$

and can easily be related to the CRLB [9]:

$$PDOP = \sqrt{tr\left\{\mathbf{J}_{\hat{\mathbf{x}}}^{-1}\right\}}\,, \quad (6)$$

where tr stands for the trace operator. In this context, two other expressions are practically relevant: The Horizontal Dilution of Precision ($HDOP$) describes the translation of measurement errors on the horizontal accuracy, i.e. the $\{x,y\}$-plane, and the Vertical Dilution of Precision ($VDOP$) describes the translation on the vertical accuracy, i.e. the z-direction. If the diagonal elements of $\mathbf{J}_{\hat{\mathbf{x}}}^{-1}$ are denoted by J_{xx}^{-1}, J_{yy}^{-1}, and J_{zz}^{-1}, these expressions are defined as follows [4, p. 268]:

$$HDOP = \sqrt{J_{xx}^{-1} + J_{yy}^{-1}} \text{ and } VDOP = \sqrt{J_{zz}^{-1}}\,. \quad (7)$$

Of course, the mathematical analysis could purely be based on the Fisher information matrix, for practical applications however, the use of the well known $xDOP$ expressions is quite convenient.

The Fisher information matrix $\mathbf{J}_{\hat{\mathbf{x}}}$ for $\hat{\mathbf{x}}$ and the given model is known as ([9, 16])

$$\mathbf{J}_{\hat{\mathbf{x}}} = \sigma_R^{-2} \cdot \mathbf{B} \cdot \mathbf{M} \cdot \mathbf{B^T}, \text{ where} \quad (8)$$

σ_R^2 can be interpreted as the mean-squared ranging error, which is a function of signal power and frequency,

$$\mathbf{B} = \left[\mathbf{b_1} - \bar{\mathbf{b}}, \mathbf{b_2} - \bar{\mathbf{b}}, \cdots, \mathbf{b_N} - \bar{\mathbf{b}}\right] \quad (9)$$

is the matrix of source/sensor bearing vectors

$$\mathbf{b_i} = [x_i, y_i, z_i]^T = \frac{\mathbf{x_i} - \mathbf{x}}{d_i} \quad (10)$$

differenced by

$$\bar{\mathbf{b}} = \sum_{i=1}^{N} m_i \mathbf{b_i} \Big/ \sum_{i=1}^{N} m_i\,, \quad (11)$$

which is the weighted average of source/sensor bearing vectors, and $\mathbf{M} = \mathrm{diag}(m_1, m_2, \ldots, m_N)$ is the diagonal matrix of all attenuation factors.

We now briefly review the optimal array geometry under the condition that the $PDOP$ is minimized, i.e. that for given measurement errors, the influence of the geometrical constellation is minimized. In [12] a nice solution has been derived with two necessary and sufficient conditions under the assumption that $m_i = 1, \forall i$:

C1: $\bar{\mathbf{b}} = \mathbf{0}$ and C2: $\mathbf{B} \cdot \mathbf{B}^{\mathbf{T}} = (N/3) \cdot \mathbf{I}$,

i.e. matrix $\mathbf{B}$ has orthogonal row vectors with equal row norm. From these conditions follows, that the optimal three-dimensional array geometry must consist of vectors $\mathbf{b_i}$, which form an uniform angular array, meaning that they are “equally” distributed on a unit spherical surface with the source located in the center. As [12] points out, the five solutions are the Platonic solids tetrahedron, octahedron, cube, icosahedron, and dodecahedron; depending on the number of sensors. Note that in these constellations the vectors $\mathbf{b_i}$ could be scaled to any length $\alpha \cdot \mathbf{b_i}$, $\forall \alpha \in \mathbb{R}^+$ and the optimum will still be achieved.

Since the solution is based on the model in (2), the effect of multipath is not included. Also, since we assume that the ranging accuracy σ_R^2 is equal for all sensors, signal power and frequency is assumed to be equal at all sensors. However, the authors do not see these as limitations to the solutions, since in this paper only the influence of the geometry should be analyzed. When necessary, special care is taken in the following.

III. Practical Considerations

The goal is now to verify and adapt the theoretical results under practical considerations. The regarded scenarios are localization applications for typical indoor environments, e.g. offices, inventory stores, production plants. To install a three-dimensional localization system in these areas, the placement of the sensors must be carefully selected. Especially, the choice of the sensors’ heights is crucial to have good resolution possibility in the z-direction. How important this is can be easily shown for the case, that four sensors would be intuitively placed in the four top corners at the ceiling of a room. In this planar configuration the $VDOP$ goes to infinity, which needs to be interpreted as the impossibility to resolve the z-coordinate, even in the case of very small measurement errors.

Therefore, we use the theoretical optimum as a basis for the “best” placement of such sensors. In the following, the number of sensors is restricted to $N = 4$. This corresponds to the minimal number of sensors in a TDOA setup with the capability to resolve three-dimensions.

In section II, the optimal solution has been derived for the origin of all vectors $\mathbf{b_i}$, i.e. the center of the tetrahedron. However, no statement has been made about other positions within the tetrahedron. Yet for practical applications, “every” position in a desired area should have a “good” localization possibility as it is not applicable to rearrange the sensors when the source it moving. Therefore, it is investigated how accurate such an optimally arranged localization system can perform. Formulas (7) and (8) were used to calculate $HDOP$ and $VDOP$ at different positions within a cube, which is spanned by the sensors in a tetrahedron arrangement. Some results are shown in fig. 1 and fig. 2, where the $HDOP$ and $VDOP$ are plotted parallel to the $\{x,y\}$-plane at different heights. As it can be seen, measurement errors are almost equally translated to position errors at all locations of the area. A complete simulation with a tight, three-dimensional grid of positions confirmed that the $HDOP$ does not exceed 2.5 and the $VDOP$ does not exceed 1.8 at any position (except of singularities at the locations of the sensors, which is regarded as a practically irrelevant). This shows that the tetrahedron constellation is a good choice for installing sensors for accurate, three-dimensional localization.

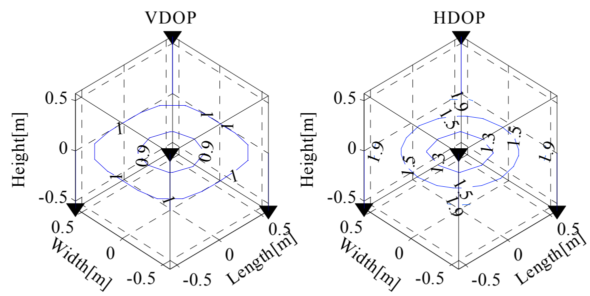

Fig. 1. Contour plots of $VDOP$ and $HDOP$ in the middle of a cube spanned by a tetrahedron constellation of sensors (triangles).

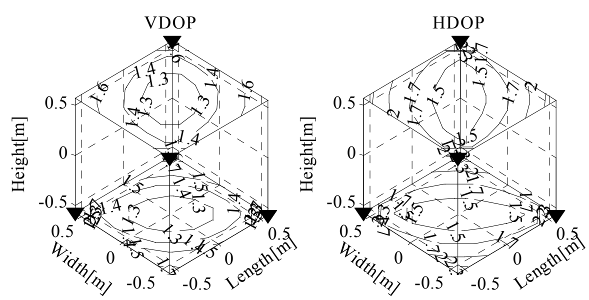

Fig. 2. Contour plots of $VDOP$ and $HDOP$ at top and bottom of a cube spanned by a tetrahedron constellation of sensors (triangles).

IV. Measurement Setup

Measurements using an Ultra-Wideband (UWB) localization system have been performed to experimentally verify the good performance of the tetrahedron geometry as well as the applicability of the simulation environment regarding reality. UWB has been chosen as technology, since it is capable of offering decimeter accuracy in ranging. The measurement setup is shown in fig. 3. A Matlab PC, which serves as a central controller and processing unit, triggers the mobile unit, which emits a bi-phase, pseudo-random-noise UWB pulse train. Four antennas are connected to a digital sampling oscilloscope, which operates as receiver. A detailed description of the system can be found in [20].

The received signal waveforms $s_i(t)$ are crosscorrelated with each other for three-dimensional positioning. Time differences are obtained by taking the argument of the peak of the crosscorrelation functions between two respective base stations and are regarded as the TDOA estimate $\Delta\hat{d}_{ij}$. At every measured position, line of sight paths between source and sensors were present. Then, Bancroft's algorithm is used as a direct solution solving the hyperbolic localization problem [15]. It has been shown, that this method sits slightly above the CRLB [21].

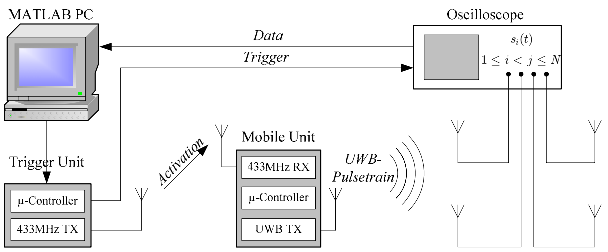

Fig. 3. UWB measurement setup using a sampling oscilloscope and a Matlab PC as a digital receiver.

Measurements were conducted in an industrial environment. The source was moved at height $0.86\mathrm{m}$ in $10\mathrm{cm}$ steps along a line with length $11.90\mathrm{m}$, resulting in $L = 100$ recorded positions. The line is indicated as an arrow in fig. 4 and fig. 5.

Measurements were carried out in two geometrical scenarios. In the first one, representing an "as good as possible" configuration, four antennas were installed at the heights of $[z_1\ z_2\ z_3\ z_4] = [3.25\ 0.88\ 3.18\ 0.82]m$, which corresponds to a "squeezed" tetrahedron geometry. The heights and the mounting positions in the $\{x,y\}$-plane were mainly determined by the area geometry. Although the tetrahedron is strongly deformed, simulation results in fig. 4 show that both, the vertical and horizontal dilution of precision at a height of $0.86\mathrm{m}$ are still quite acceptable. Hence, with a decimeter ranging accuracy, the horizontal position accuracy would be about two, the vertical about five decimeters.

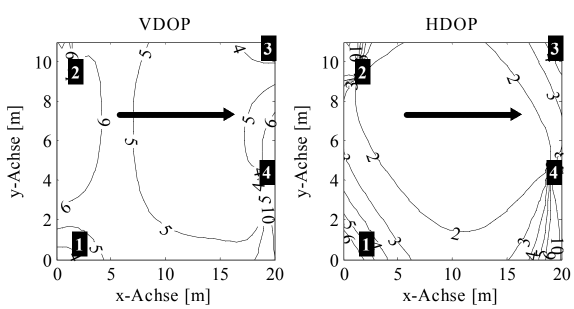

Fig. 4. Contour plot at height of 0.86m with a "squeezed" tetrahedron constellation of sensors (represented by the numbered squares)

The second geometry follows the more "intuitive" approach to install all sensors at the ceiling of the room in an almost planar configuration. This is supported by the fact that most of the times an installation like this will lead to less non-line-of-sight situations, which would also be preferable for accurately estimating ranges. Also, in a typical indoor environment an installation of the sensors can be difficult at lower heights due to machinery, shelves, etc. The respective heights of the antennas are $[z_1\ z_2\ z_3\ z_4] = [3.25\ 3.25\ 3.18\ 3.15]m$.

The simulations in fig. 5 show that the vertical dilution of precision increases quite badly, even though the $HDOP$ remains still acceptable in most of the areas.

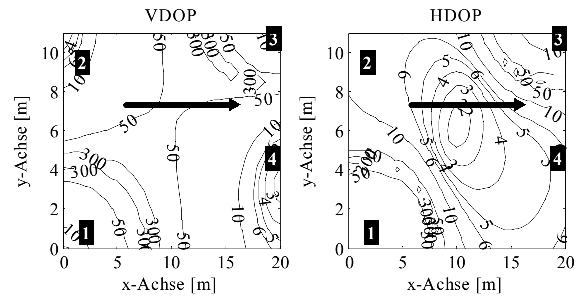

Fig. 5. Contour plots of $VDOP$ and $HDOP$ at height of 0.86m with an almost planar constellation of sensors (represented by the numbered squares)

V. Results and Discussion

To compare the simulated results with the measured ones, an estimate for the $xDOPs$ at position $\mathbf{x}_l$ must be calculated from the range measurements using (5) and (7). Therefore, the RMS range measurement error at position $\mathbf{x}_l$ is estimated as follows:

$$e^r_{\mathbf{x}_l} = \sqrt{\sum_{i=1}^{3}\sum_{j=2}^{4}\left(\Delta\hat{d}_{ijl} - \Delta d_{ijl}\right)^2 / 6}\,, \tag{12}$$

with $\Delta\hat{d}_{ijl}$ being the measured and Δd_{ijl} the real range difference at position $\mathbf{x}_l$.

The horizontal RMS position error is estimated as follows:

$$e^h_{\mathbf{x}_l} = \sqrt{(\hat{x}_l - x_l)^2 + (\hat{y}_l - y_l)^2}\,, \tag{13}$$

the vertical RMS position error $e^v_{\mathbf{x}_l}$ similarly. Both errors of all recorded measurements are depicted in fig. 6. The degradation in vertical direction of the planar configuration can easily be observed.

To gain a statistical basis, an arithmetic mean of all $L = 100$ $xDOPs$ is used as an approximation for the influence of the geometrical constellation over the complete measurement line:

$$HDOP = \frac{1}{L}\sum_{l=1}^{L}\frac{e^h_{\mathbf{x}_l}}{e^r_{\mathbf{x}_l}} \text{ and } VDOP = \frac{1}{L}\sum_{l=1}^{L}\frac{e^v_{\mathbf{x}_l}}{e^r_{\mathbf{x}_l}}. \tag{14}$$

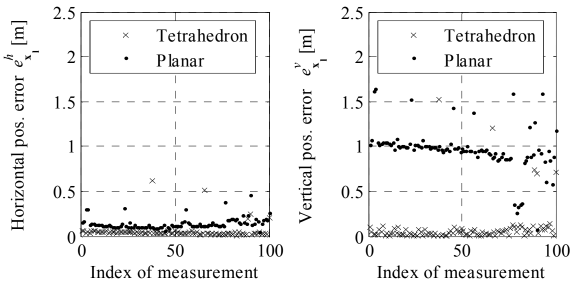

Fig. 6. Position estimation errors in horizontal (left) and vertical (right) direction of each recorded position.

Finally, to compare the simulated with the estimated results, the power and frequency dependence of σ_R^{-2} is removed, by using a ratio expression for the $xDOPs$:

$$HDOP_{ratio} = \frac{HDOP_{planar}}{HDOP_{tetrahedron}}. \quad (15)$$

The estimate of the $VDOP_{ratio}$ is calculated similarly. Note, that such a comparison is only valid if the summarized RMS ranging error of both, the tetrahedron and the planar constellation, are comparable. In fact, this is the case with $e_{RMS}^{planar} = 0.113\text{m}$ and $e_{RMS}^{tetrahedron} = 0.123\text{m}$.

All $xDOP$ ratios are summarized in Table I, and two conclusions can be drawn. On the one hand, the measurements show that the simulations, which are based on theoretical derivations, are a good approximation of the reality. On the other hand, the strong deterioration of the vertical position estimation is based on the almost planar arrangement of the sensors.

TABLE I
COMPARISON OF SIMULATED AND MEASURED XDOP-RATIOS

	$VDOP_{ratio}$	$HDOP_{ratio}$
SIMULATED	10,4	2,7
MEASURED	12,3	2,5

VI. CONCLUSION

The practicability of previously derived optimal geometries for three-dimensional TDOA localization has been investigated. With simulations and real-world measurements in typical indoor scenarios it has been shown, that the optimal solutions can be “bent” and still are quite applicable. Although the results can be applied for installing three-dimensional indoor localization systems, the tradeoff between ranging accuracy and geometry has always to be taken care of: striving to arrange the sensors as good as possible in a tetrahedron can lead to non line of sight situations, which degrade the ranging accuracy and therefore inherently deteriorate the position accuracy. However, the geometrical implications of different situations can well be simulated and different scenarios can be evaluated to find a good geometrical setup for three-dimensional localization.

REFERENCES

[1] P. Schultheiss and E. Weinstein, "Lower bounds on the localization errors of a moving source observed by a passive array," IEEE Transactions on Acoustics, Speech, and Signal Processing vol. 29, pp. 600-607, 1981.

[2] W. R. Hahn, "Optimum signal processing for passive sonar range and bearing estimation," The Journal of the Acoustical Society of America, vol. 58, pp. 201-207, 1975.

[3] E. Weinstein, "Optimal source localization and tracking from passive array measurements," IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 30, pp. 69-76, 1982.

[4] E. D. Kaplan, Understanding GPS : principles and applications. Boston [u.a.]: Artech House, 1996.

[5] B. Hofmann-Wellenhof, H. Lichtenegger, and J. Collins, Global positioning system : theory and practice, 5., revised edition ed. Wien [u.a.]: Springer, 2001.

[6] M. A. Spirito, "Accuracy of hyperbolic mobile station location in cellular networks," Electronics Letters, vol. 37, pp. 708-710, 2001.

[7] A. B. Gershman and J. F. Bohme, "A note on most favorable array geometries for DOA estimation and array interpolation," IEEE Signal Processing Letters, vol. 4, pp. 232-235, 1997.

[8] C. T. Chambers, T.C.; Sharman, K.C.; Durrani, T.S., "Temporal and spatial sampling influence on the estimates of superimposed narrowband signals: when less can mean more," IEEE Transactions on Signal Processing, vol. 44, pp. 3085-3098, 1996.

[9] J. S. Abel, "Optimal sensor placement for passive source localization," Proceedings of International Conference on Acoustics, Speech, and Signal Processing, ICASSP, Albuquerque, USA, 1990, pp. 2927-2930.

[10] H. Zhang, "Two-dimensional optimal sensor placement," IEEE Transactions on Systems, Man and Cybernetics, vol. 25, pp. 781-792, 1995.

[11] T. Hegazy and G. Vachtsevanos, "Sensor Placement for Isotropic Source Localization," in Lecture Notes in Computer Science, Information Processing in Sensor Networks: Second International Workshop (IPSN) ed. Palo Alto, USA: Springer, 2003, pp. 432-441.

[12] B. Yang and J. Scheuing, "Cramer-Rao bound and optimum sensor array for source localization from time differences of arrival " Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Philadelphia, USA, 2005, pp. 961-964.

[13] D. B. Jourdan and N. Roy, "Optimal Sensor Placement for Agent Localization," Position Location and Navigation Symposium (PLANS), San Diego, USA, 2006.

[14] B. T. Fang, "Simple solutions for hyperbolic and related position fixes," IEEE Transactions on Aerospace and Electronic Systems, vol. 26, pp. 748-753, 1990.

[15] S. Bancroft, "An Algebraic Solution of the GPS Equations," IEEE Transactions on Aerospace and Electronic Systems, vol. AES-21, pp. 56-59, 1985.

[16] Y. T. Chan and K. C. Ho, "A simple and efficient estimator for hyperbolic location," IEEE Transactions on Signal Processing, vol. 42, pp. 1905-1915, 1994.

[17] S. M. Kay, Fundamentals of statistical signal processing : estimation theory. Englewood Cliffs, NJ: PTR Prentice-Hall, 1993.

[18] H. B. Lee, "A Novel Procedure for Assessing the Accuracy of Hyperbolic Multilateration Systems," IEEE Transactions on Aerospace and Electronic Systems, vol. AES-114, No. 1, pp. 2-15, 1975.

[19] D.-H. Shin and T.-K. Sung, "Comparisons of error characteristics between TOA and TDOA positioning," IEEE Transactions on Aerospace and Electronic Systems, vol. 38, pp. 307-311, 2002.

[20] J. Schroeder, S. Galler, and K. Kyamakya, "A Low-Cost Experimental Ultra-Wideband Positioning System," Proceedings of IEEE International Conference on Ultra-Wideband, Zürich, Switzerland, 2005, pp. 632-637.

[21] J. Abel and J. Chaffee, "Direct GPS Solutions," 49th ION Annual Meeting Proceedings, pp. 457-466, 1993.