Bifurcation analysis and synchronization issues in a short life cycle products supply chain

K.R. Anne, J.C.Chedjou, K. Kyamakya

***Abstract*— In today's global market-place, the majority of products have a short product life cycle due to the innovative character of the products and ever changing customer desires. This short product life cycle nature produces various types of uncertainties along the supply chain, e.g. demand uncertainty, supply uncertainty, delivery uncertainty and forecasting uncertainty. These uncertainties make supply chains complex and nonlinear systems as they propagate along the supply chain in both upstream and down stream. This work investigates the dynamical behavior of a three-echelon supply chain. The modeling of this structure is carried out to display its nonlinear dynamical behavior. It is shown that the dynamics (e.g. stability) of the supply chain is very sensitive to external uncertainties. Specifically, the supply chain subjected to these uncertainties can exhibit strange and undesired dynamics/states such as saturation and chaos. An adaptive algorithm for the automatic cancellation of these strange dynamics due to uncertainties is developed by re-adjusting the internal parameters of the supply chain in order to achieve its synchronization. A bifurcation analysis is also carried out. This analysis is essential and useful for strategic decision makers as it allows both the visualization and control of the states/dynamics of the entire supply chain. In order to display the dynamics of a real world supply chain, the bifurcation analysis is performed for the Tamagotchi™ supply chain.**

***Index Terms*— Supply chain synchronization; Supply chain control; Bullwhip effect; Chaos in supply chains; Bifurcation analysis for supply chains.**

I. Introduction

SUPPLY chain network management has been defined as the management of upstream and downstream relationships with suppliers and customers in order to create enhanced value in the final market-place at less cost to the supply chain as a whole [1]. Today's global market-place is increasingly dynamic and volatile. This dynamic and volatile nature produces various types of uncertainties along the supply chain e.g., demand uncertainty, supply uncertainty, delivery uncertainty, forecasting uncertainty. Apart from these uncertainties caused by external sources, uncertainties observed on daily basis (e.g. machine breakdowns, wrong supplies, supply shortages etc.) make supply chains much more complex systems. The uncertainties propagate along the upstream and downstream of the entire supply chain leading to the production of various nonlinear dynamic effects. In addition to these uncertainties, the relation between the various players in the supply chain is often characterized by the mistrust and competition.

In fact, inventory is generally used as insurance against the uncertainties. In the case of a single enterprise based supply chain it is relatively easy to overcome the uncertainties with properly sized inventories at each stage like raw materials, work in process and finished goods inventories. The present day statistical tools and forecasting methods can satisfactorily aid in determining how much must be hold to satisfy the customer demand for the particular product despite the uncertainties [2]. However, the problem is much more complicated when considering the whole network consisting of different players distributed globally. Nearly each player holds some inventory to protect against uncertainties, but the real difficulty is in determining how much must be hold and where to hold it. To date, there is no clear analytical way to calculate the propagation of uncertainties up and/or down the supply chain. Traditionally, firms have relied on the experience and intuition in facing uncertainties.

The decisions made with intuition can make the supply chain exhibit various dynamic states like chaos. Chaos is defined as an aperiodic, unpredictable and bounded dynamics in a deterministic system with sensitivity dependence on initial conditions. Chaos is a disorderly long term evolution occurring in deterministic nonlinear systems. The beer game developed at MIT to introduce the students and industrialists to the concepts of economic dynamics shed further light on supply chain dynamics [3]. It has been found that one in four management teams in the supply chain creates deterministic chaos in the ordering pattern and inventory levels [4]. This clearly demonstrated in practice the occurrence of chaos in supply chains.

The objectives of this paper are: (1) to show the potentials of the concepts of nonlinear dynamics and modelling in supply chain networks; (2) to show the interest of synchronization in supply chain networks; (3) to offer to strategic decision makers an approach or technique to control and stabilize the states of their supply chain networks when subjected to uncertainties; and (4) to demonstrate the dynamics of a Tamagotchi(T) supply chain through bifurcation analysis.

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II. Responding to Uncertainty

The objective of any player in a supply chain network is to achieve maximum profits and give maximum customer satisfaction. In addition to the stated individual objective all players also have a responsibility towards the global objective of resilient Supply Chain Networks (SCN).

Christopher [5] emphasises this by stating:

Competition in the future will not be between individual organizations but between competing supply chains

This realization has made the entities in SCN to look beyond their own boundaries to asses how the resources of each other can be utilized to achieve the global objective without compromising on their own objective. In the midst of pursing towards it, entities have relentlessly restructured and reengineered their internal organizational boundaries and policies with an objective of transforming their relations from "arm's – length" relationship to "durable arm's – length" relationships [6].

The definition of a truly efficient supply chain is when all players involved are communicating correct data – a manufacturer is communicating the correct product information and receiving accurate purchase orders; a retailer is receiving the specific products that were ordered; and the product is available to the end consumer at the right time and at the right price. However, communicating the correct data is always not possible in supply chains as each stake holder has different objectives and constrains. The traditional supply chain management has been based on limited information sharing restricted to the product in consideration and transaction oriented towards that product [7]. It is well studied [2, 8-10] that information sharing, demand pattern, ordering policy, and lead time have a direct impact on the performance of supply chains. Information sharing can reduce the lead time. Lead time reduction is found to be very beneficial and can reduce inventory and demand variability and improve customer service [8].

However, during the process of identifying the ways to mitigate the effects due to uncertainties, companies within the supply chain realized that they need to achieve the self organization of the supply chain they belong to in order to satisfy the stated objective. In accordance with the greater focus on the self-organization of the supply chain, the companies are increasingly focusing on the pre-requisitions like integration, collaboration and synchronization between all entities in the supply chain as shown in Figure 1.

The first step towards self-organized supply chains is the integration stage. A Complex corporate- structure demands the various functional units within a company to be integrated first. The intra-entity integration also called the functional integration is the basic driver towards the integration of the entire supply chain. The functional integration of purchasing, manufacturing, transportation and warehousing activities gives the very much needed visibility to the supply chain. Besides the functional integration, the inter-temporal integration also called hierarchical integration of these activities over strategic, tactical, and operational levels is also important [11-13]. This

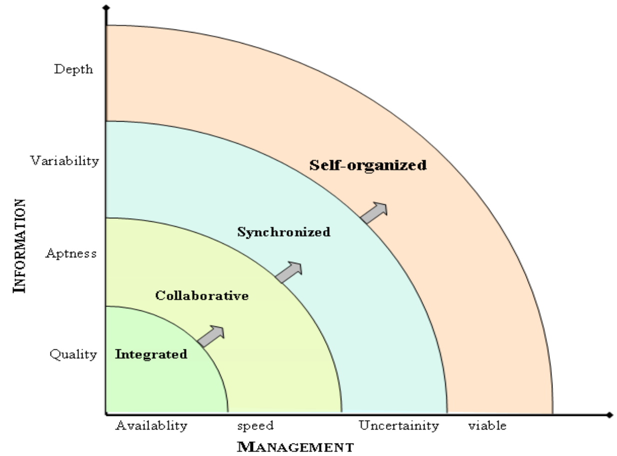

Figure 1. Key stages to the evolution of adaptive supply chain networks

inter-temporal integration requires consistency among overlapping of supply chain decisions at various levels of planning. However, the major role of the inter-temporal integration is in designing the supply chain for the product. The improved integration of activities across multiple companies/entities in a supply chain allows the collection of data from the breadth of the supply chain [14]. Information technology is the key enabler for the integration in supply chains. The information systems like EDI (Electronic Data Interchange) and ERP (Enterprise Resource Planning) provide the necessary information needed for the integration [5]. The information systems make the information available; however the effect of integration directly depends on the quality of the information made available.

After the integration of the supply chain, the next step towards self-organization is the collaboration between entities. The objective of the collaboration is to make the information available when it is needed. Of course collaboration concepts are not new; they exist from the days of traditional business in the form of contracts. However, the effectiveness, execution speed, of the collaborations is highly increased due to the technological advances and the integration tools. Collaboration is strong where business to business relationships are strong. The degree of collaboration varies depending upon the strength of the integration [15]. But true collaboration among and in between all the entities in a SCN is more difficult in practice.

Even though data is made available with the help of integration tools and collaborative agreements, often the collected data paints a false image of an operation due to data entry errors and inconsistent collection procedures. Further, the upstream/downstream requirements are sometimes not clearly understood, not accurate enough and are not up-to-date. The breadth of the supply chain can compound the accuracy problem as the data can be re-worked or re-created in between.

In order to cope up with inaccurate and/or varied data, synchronization is an important step in dealing with uncertainties. Synchronization can be classified into two types, namely the complete synchronization and the partial synchronization. The complete synchronization is observed when the data synchronization is achieved, leading to the real-time access to available data by all players at the same time. Complete synchronization enables the supply chain to react quickly to changes in demand and in product design. This type of synchronization is particularly suitable in just-in-time supply chain networks. To achieve the complete synchronization the complete chain should follow the integration and collaboration methods in true spirit.

Partial synchronization is achieved through a feedback controller item. Apart from the data synchronization as explained in complete synchronization, a controller item is developed to mitigate the effects due to both inaccurate data and uncertainties. In this type of synchronization the major effort is placed in quantifying the effects due to uncertainties. The modelling and quantification of the effects caused by the time lag (i.e. time delay), the information discrepancy, and the individual objectives help in designing the controller/synchronizer element. This controller item can be unique for each entity or supply chain as a whole. Uncertainties and exceptions are identified early and the data for intelligent response are immediately available. This greatly minimizes the bullwhip effect, demand amplification, and saves downstream partners and customers from needless activity [10, 11, 16, 17]. In this paper, we propose an adaptive controller to achieve synchronization phenomena in order to mitigate the effects due to uncertainties.

III. Modeling of a Three Level Supply Chain

The theoretical framework for supply chain management underlies the setting, optimization and control of the system model. The system model is not unique for all the supply chains [18]. The system dynamics change for each type of product e.g. food, oil, consumer goods, etc., depending upon the processes involved. The system dynamics based approach to model the business dynamics was first introduced by Forrester [19]. The system dynamics has its origins in control engineering and management. The approach uses a perspective based on information feedback and delays to understand the dynamic behaviour of complex physical and social systems. System dynamics is an approach which is actively used to model the managerial behaviour.

In 1989, Sterman [3] proposed a model that can be used to analyse the supply chains using the industrial dynamics fundamentals proposed by Forrester. The Sterman model was actively used to analyse the supply chain dynamics. Due to its visual nature and simplicity, after a lengthy floppy period, the system dynamics approach is gaining momentum in modelling the inventory management process, the policy development and demand amplification [20]. The system theory based modelling is also used to develop the feedback controllers to mitigate the bullwhip effect (i.e. the demand amplification) to some extent [21]. However, to address the issues emanating from uncertainties and the problems occurring in real time, the system dynamics modelling might not be suitable [8, 20, 21].

In recent years, many researchers are also using an agent based distributed modelling approach to cope with supply chain networks (SCN) [13, 22, 23]. One or several agents can be used to represent each entity in the SCN. Each agent is assigned with a local objective and global objective as well. With the advent of mobile agents which can run on lighter platforms, the use of agents to both collect information and take decisions has become popular. Moreover, the agent paradigm is a natural metaphor for network organizations, since companies prefer maximizing their own profit rather than the profit of the supply chain [22]. The multi agent based modelling approach offers a way to elaborate the supply chain as the agents are autonomous and the agent rules can be defined in advance. The distributed decision nature of the multi agent systems makes it easier to add other entities in the local environment. Entities leaving the SCN in the middle will not affect the entire SCN to a great extent. The agent rule framework provides certain amount of trust among the partners as it eliminates the mistrust and deception among entities.

The major disadvantages with the multi agent based SCN modeling is that, as there is no global view of the system theoretical optimisation, the optimization of the supply chain can't be visualized. As per the system dynamics theory any system can be unstable [19]; this theorem proves that the multi agent system, which is a system with multi autonomous elements/entities, again can be unstable. Further, the multi agent system is also based on the assumption that all the participating entities in the SCN are truly integrated and

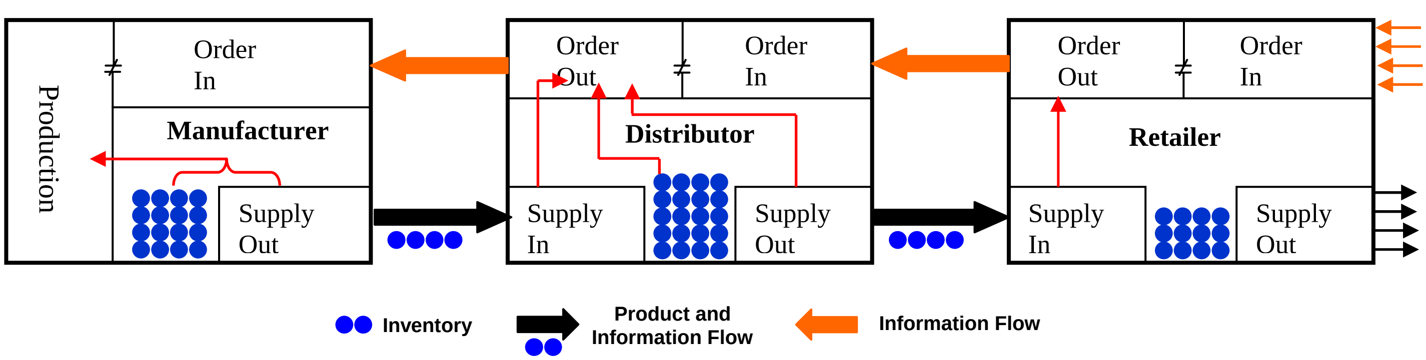

Figure 2. Model of a three echelon supply chain

collaborating. However “true integration and collaboration” is highly difficult [3, 10, 17, 24-26].

By considering these issues (in this work) we envisage the complex nonlinear modelling of a three echelon supply chain to represent the realistic dynamics. A three level model is envisaged (Figure 2) to describe a simple scenario in a very complex supply chain. The nonlinear supply chain models in the literature [18, 27] focus mainly on the specific tasks, and thus becomes a transaction oriented approach. In this work, we mainly focus on building a nonlinear supply chain model that can exhibit more complexity covering the information distortion, retailer order satisfaction and safety stock. An additional criterion is the extreme sensitivity of the model to both uncertainties and initial conditions. The following notations are introduced to facilitate the description of the model:

i	Time period
m	Rate of customer demand satisfaction at retailer
r	Rate of information distortion of products demanded by retailer
k	Safety stock coefficient at manufacturer
x_i	The quantity demanded by retailer in current period
y_i	The quantity distributors can supply in current period
z_i	The quantity produced in current period depend on the order

The orders they make might not be equal to orders they receive. The order-out quantity depends not only on how much inventory you have already, but also how much you want to supply out. The order-out quantity at retailer depends on the ratio m at which the demand is satisfied during the previous order. The distributor needs to take into consideration among other things, the rate of information distortion r that can occur in the received orders. The producer needs to take care about the safety stock k in order to avoid the small production batches. These scenarios/phenomena are described in Figure 2. An in-depth explanation is provided below and a corresponding mathematical model is derived to analyse the dynamics of the SCN.

We consider that the demand information is transmitted within the layers of the supply chain with a delay of one unit time. As illustrated in Figure 1, the ordering quantity is not the same as the requested order quantity at any level. The order quantity at the current period of time at retailer is linearly coupled with the distributor and it is influenced by how much of demand is satisfied in the previous period of time. This scenario/phenomenon is modelled by Eq. (1).

$$x_i = m(y_{i-1} - x_{i-1}) \tag{1}$$

Here m is the ratio at which the demand is satisfied. The dependency/coupling between the distributor, the producer and the retailer (Figure 2) is not linear. Indeed the distributor needs to take the combined effect of retailer and producer into consideration before making his order, i.e., quadratic coupling. Apart from this, the distributor also needs to take into consideration that the order information received from the retailer might be distorted. This scenario is modelled by Eq. (2).

$$y_i = x_{i-1}(r - z_{i-1}) \tag{2}$$

Here, r is the information distortion coefficient. The production quantity from the producer unit typically depends on the distributor's orders and the safety stock. However the distributors' orders again depend on the retailer's orders, i.e., the producer needs to take the combined effect of retailer and distributor into account before making production decisions. This scenario is modelled by Eq. (3)

$$z_i = x_{i-1}y_{i-1} + kz_{i-1} \tag{3}$$

Eqs. (1) – (3) represent the quantity demanded by customers (Eq. (1)), the inventory level of distributors (Eq. (2)) and the quantity produced by producers (Eq. (3)), where:

$x_i < 0$ denotes that the supply is less than customers demand in the previous period

$y_i < 0$ denotes that the information is severely distorted and no adjustment is necessary at the inventory level

$z_i < 0$ denotes the cases of overstock or return and hence no new productions.

Eqs. (1) - (3) are discrete models describing the dynamics of the SCN of Figure 2. Considering very small time intervals, the continuous model in Eq. (4) can be derived from Eqs. (1) - (3).

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$$\begin{cases} \dot{x} = my - (m+1)x \\ \dot{y} = rx - y - xz \\ \dot{z} = xy + (k-1)z \end{cases} \tag{4}$$

If the conditions $\sigma = m + 1$ and $b = 1 - k$ are satisfied, Eq. 4 leads to the *Lorenz* model in Eq. 5.

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = rx - y - xz \\ \dot{z} = xy - bz \end{cases} \tag{5}$$

From the theory of dynamic systems it is proved that this model produces a wide variety of nonlinear features depending upon the parameters values. This model is particularly of interest when dealing with the modelling of scenarios/phenomena which are very sensitive to initial conditions and to uncertainties as well. In the concrete case of SCN, uncertainties, when added at one layer effectively propagate in both upstream and down stream. This is a common dynamics exhibited by realistic SCN. The similar model (Eq. (5)) is also proposed by the authors in [17] to exhibit the supply chain dynamics and mitigate the bullwhip effect. In this work, we are considering the external uncertainties caused by external perturbations. These perturbations can occur due to the market-place dynamics and volatility. We consider the external perturbations are considered to be nonlinear as the market-place behaviour is nonlinear in nature. Assuming that the perturbations can affect any of the three levels of the SCN, a perturbed form of Eq. (5) is proposed in Eq. (6).

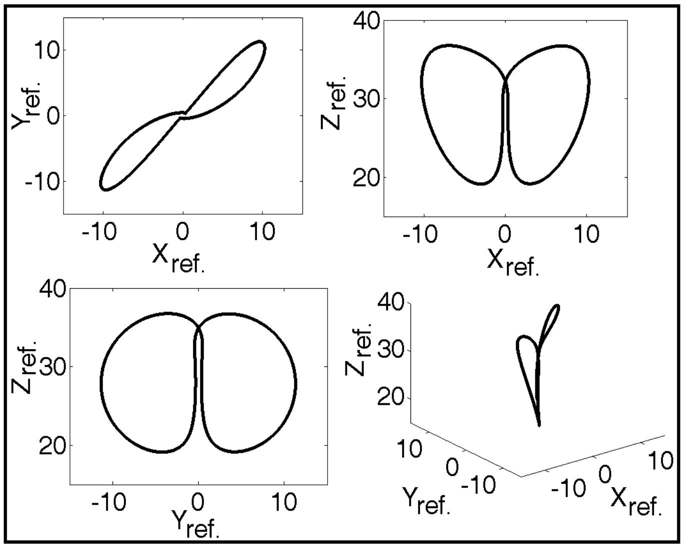

Figure 3. Phase space representation of the reference supply chain for $\sigma = 15$ $r = 29$ and $b = 2/3$

$$\begin{cases} \dot{x}' = \sigma(y' - x') + d_1 \\ \dot{y}' = rx' - y' - x'z' + d_2 \\ \dot{z}' = x'y' - bz' + d_3 \end{cases} \qquad (6)$$

Where $d_i \, (i = 1,2,3)$ represent the external perturbations. Before considering the effects of external perturbations on the supply chain we define the reference model with the following parameters values $\sigma = 15$ $r = 29$ and $b = 2/3$. These values illustrate the regular state of the reference supply chain model. The phase space structure of the reference model is shown in Figure 3.

After defining the reference model we analyse the effects of external perturbations on this model. Basically we are concerned with the new dynamics exhibited by the reference supply chain subjected to external perturbations.

IV. Synchronization of an Externally Perturbed Supply Chain

In this section, we briefly discuss the traditional/classical approach to investigate synchronization issues. Modern synchronization tools provide an automation framework but do not concentrate on what happen if the given data is slightly changed accidentally. Many companies have taken inspiration from the modern web and wireless technologies to make the synchronization in a timeliness manner [11, 26]. The integration efforts and the collaboration for the processes within the SCN certainly improved communication by means of EDI and current internet based web information exchanges. Better information (point of sale data and the Collaborative Planning, Forecasting, and Replenishment, CPFR, initiatives), and a general willingness to work more closely together made the timeliness of information possible to certain extent. Nevertheless, the efficiencies have been gained through improvements that any executive can effect at his or her own workplace by putting in place the appropriate company-wide initiatives aimed at improving the internal business process. However, as we have seen the uncertainties propagate in both directions (upstream and downstream) along a SCN, network wide initiatives are necessary to mitigate the effects caused by uncertainties.

In this context, we provide different cases of perturbations affecting the data and present techniques/methods to synchronize or stabilize the new states (or perturbed states) exhibited by the SCN. The causes of instability of the supply chain can be broadly classified into two categories. The first cause is the dynamical and nonlinear character of the motions (i.e. material/products flow, information exchanges, etc.,) between different entities in supply chains. The second cause originates from the effects of both external and internal perturbations [28] to which the supply chain is subjected.

An optimal management of the information flows within the supply chains may be of high importance in order to alleviate the effects leading to negative consequences on the flows within the supply chains. This could be achieved through an adaptive control mechanism which is based on a current comparison of the dynamical data within the supply chains with the pre-defined data fixed by the requirements of the supply chains. Here, an automatic or adaptive control of the flows within the supply chains should be able to detect changes in the flows within the supply chains and act accordingly/consequently (by undertaking a given action within the supply chains) in order to alleviate the undesirable effects and therefore stabilize the system behaviour that has been perturbed. The achievement of synchronization is observed when the action undertaken has allowed the recovery of the original behaviour (eventually thresholds or reference requirements) of the supply chain. The schematic description of the adaptive synchronization of supply chain is illustrated in the Figure 4.

This paper develops an adaptive method (algorithms and/or tools) for the systematic and automatic control of the flows within the supply chains. In fact, due to the dynamic changes as discussed (in time domain), some pre-defined settings or requirements within the supply chains (thresholds like safety stocks) may be varying accordingly as consequence of these perturbations. It should be worth mentioning that a combination of the simultaneous effects of both internal and

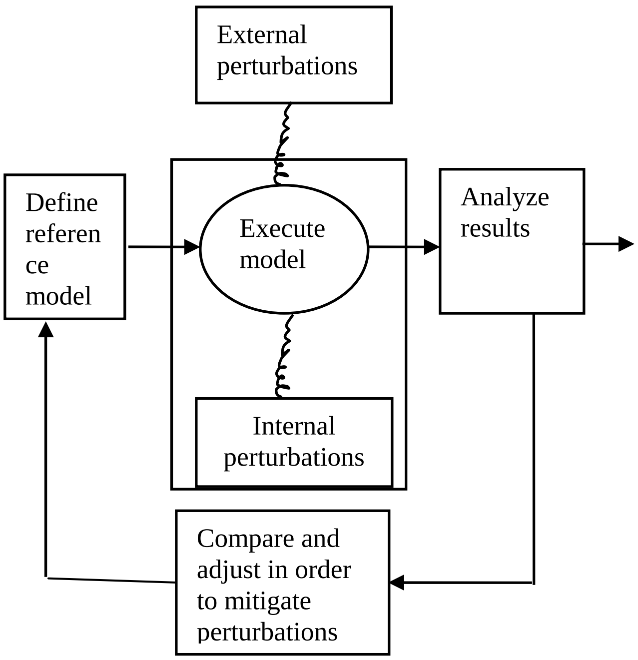

Figure 4. An adaptive feedback control model to mitigate the effects due to uncertainties and perturbations

external perturbations may be responsible of the dynamic motion variations (e.g. flow of materials, information

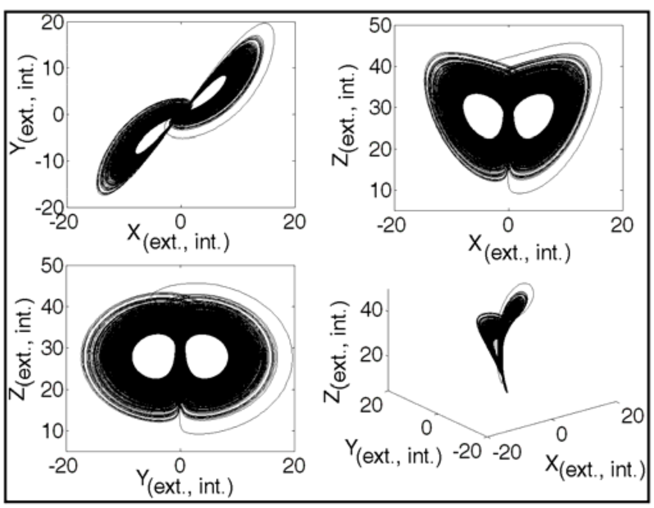

Figure 5. Chaotic state of the supply chain due to external perturbations

exchange, etc.) within the supply chain. This is a concrete and/or realistic scenario as the supply chains of many companies are currently exposed to the both types of perturbations. However the analysis in this work is restricted to the case where the reference model is subjected to external perturbations due to the fluctuation in the market demand.

Figure 5 shows a new representation of the phase space structure of the reference supply chain subjected to the following external perturbations: $d_1 = 0.56\cos(3t)$, $d_2 = 20\cos(5t)$ and $d_3 = 50\cos(1020\cos(5t)t)$.

The effects of external perturbations on the original (or specific) requirements of the reference supply chain are clearly shown by the well-known chaotic Lorenz attractors exhibited by the reference SCN subjected to perturbations. The cancellation process of these effects is achieved by exploiting the synchronization controller item shown in Figure 4. The synchronization process concerns two different models of the supply chain: (a) the reference model (unperturbed model) described by Eq.(5), and (b) the externally perturbed model described in Eq. (6). Following the active control approach of Liao [29], for the purpose of synchronization we vary the internal parameters of the perturbed supply chain. In order to vary the internal parameters, we define the state errors between the perturbed system and the reference system by Eq. (7).

$$e_x = x' - x;\ \ e_y = y' - y;\ \ e_z = z' - z; \qquad (7)$$

x', *y'* and *z'* are perturbed states and *x*, *y* and *z* are unperturbed states. The synchronization problem in this context can be equivalent to the problem of stabilizing the system shown in Eq. (7). This is possible through a suitable choice of the internal variables of the perturbed system. In fact, the adaptive control algorithm.

Considers the effects of external perturbations and adjusts the values of the internal parameters (σ, r, b) of the three echelon supply chain by tiny variations $d\sigma$, dr and db. The variation of each internal parameter is performed in well defined ranges (or windows) of variation. Performing the parameters variations in these ranges is necessary as we can't vary the parameters beyond the realistic scenario. A threshold error is fixed (which is less than approximately 0.02) under which full alleviation of the effects due to external perturbations is supposed to be effective; this leads to the achievement of synchronization, which results in the recovery of the behavior of the reference system.

Figure 5 shows the structure in phase space of the supply chain subjected to external perturbations $(d_1 = 0.56\cos(3t), d_2 = 20\cos(5t), d_3 = 50\cos(10t))$. This structure in the phase space shows the occurrence of the well- known bullwhip and chaotic effects in the three level supply chains. The regulation process has been exploited to adjust the internal parameters values in order to achieve synchronization, i.e. the full cancellation of the effects due to perturbations. The corresponding values for the achievement of the regulation process are $d\sigma = 35$, $dr = 15$ and $db = 0.09$. The results of this process are shown in Figure 6. Indeed, below the precision 2%, Figure 6 shows attractors similar to those of the reference or original supply chain system.

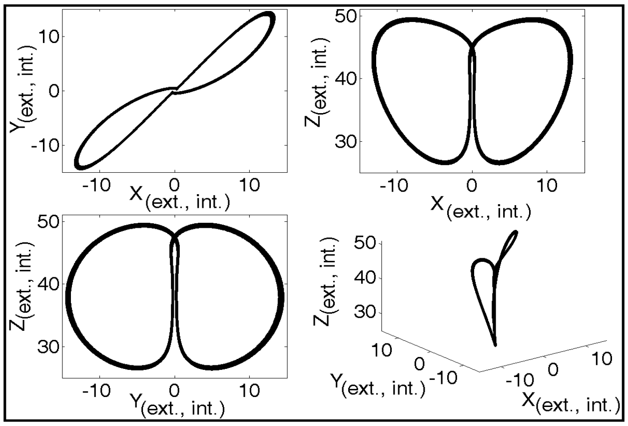

Figure 6. Alleviation of the chaotic effect caused by external perturbation with adaptive synchronization

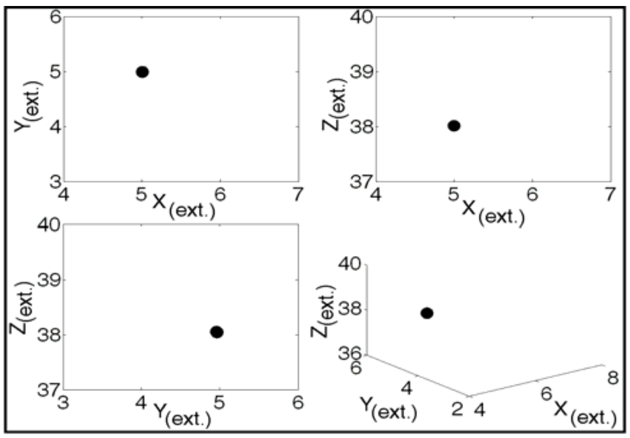

Figure 7. Saturation state of the supply chain due to external perturbations

Further investigations have been performed to show that the supply chain subjected to external perturbations can exhibit the state of saturation which is characterized by non dynamic (or fixed) data in each level of the three levels. Indeed, for $d_1 = 10\cos(5t)$, $d_2 = 5\cos(10t)$ and $d_3 = 10\cos(10t)$, the achievement of the state of saturation is clearly shown in Figure 7. The saturation manifests itself by a sudden exhibition of fixed or constant values/data along each level of the externally perturbed three echelon supply chain. When the state of saturation is obtained, further changes/flows in the supply chain are not represented effectively. To alleviate the effect (i.e. saturation) due to external perturbations, we performed the adaptive regulation process explained before by adjusting the internal parameters of the SCN. The appropriated values of the internal parameters to alleviate the effects due to external perturbations are $d\sigma = 4$, $dr = 2$ and $db = 1$. The result of the regulation process is shown in Figure 8. Indeed, tiny variations of the internal parameters of the supply chain lead to the achievement of synchronization. This is manifested by the abrupt change of the state of the system from the saturation state (Figure 7) to a regular state (Figure 8) which is similar to the state of the reference supply chain (Figure 3).

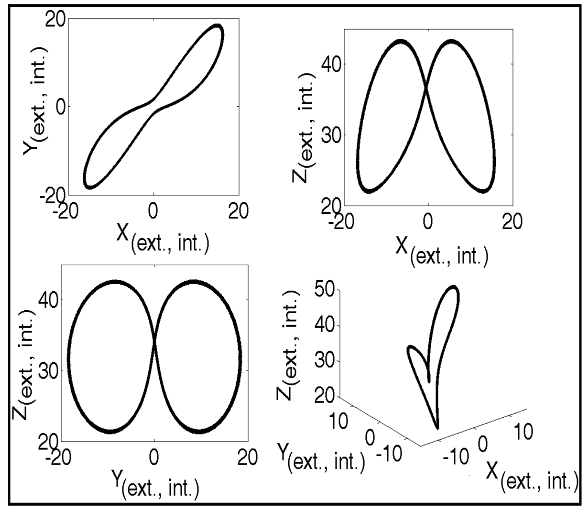

Figure 8. Alleviation of the saturation effect caused by external perturbation with adaptive synchronization

The supply chain subjected to external perturbations can exhibit various striking states such as chaos (Figure 5) and saturation (Figure 7) to name a few. The regulation process can be performed to cancel or mitigate the effects due to external perturbations. This process is based on the adjustment of the internal parameters of the supply chain. Various new and interesting states of the supply chain are discovered towards the achievement of synchronization which is characterized by the cancellation or alleviation of the effects due to external perturbations. Therefore, an interesting and open question must be concerned with the exploration of appropriated methods to control the states of the supply chain. Indeed, the stability of the supply chain is not robust (i.e. very sensitive) to external perturbations. The control process might lead to the derivation of the parameters ranges (windows) in which each of the various states of the supply chain can be found (or defined). The bifurcation analysis is an appropriate method to describe the various states of the supply chain in well specified windows of parameters.

V. Bifurcation Analysis

The bifurcation is a qualitative change observed in the behavior/state of a system as its parameters settings vary. The bifurcation is observed if the state of the system suddenly changes qualitatively upon small /smooth variation of the parameter values. The bifurcation theory [30] is the analysis/study of the bifurcation scenarios with the aim of defining/determining the states (equilibrium/fixed points, periodic or chaotic states) of the system in a given parameter space. Basically, bifurcation values/points are critical values leading to qualitative changes in the states of the system.

The preceding section has shown that the annihilation or alleviation of the effects due to external perturbations is possible through the achievement of synchronization. Nevertheless, during the regulation process, we found that the

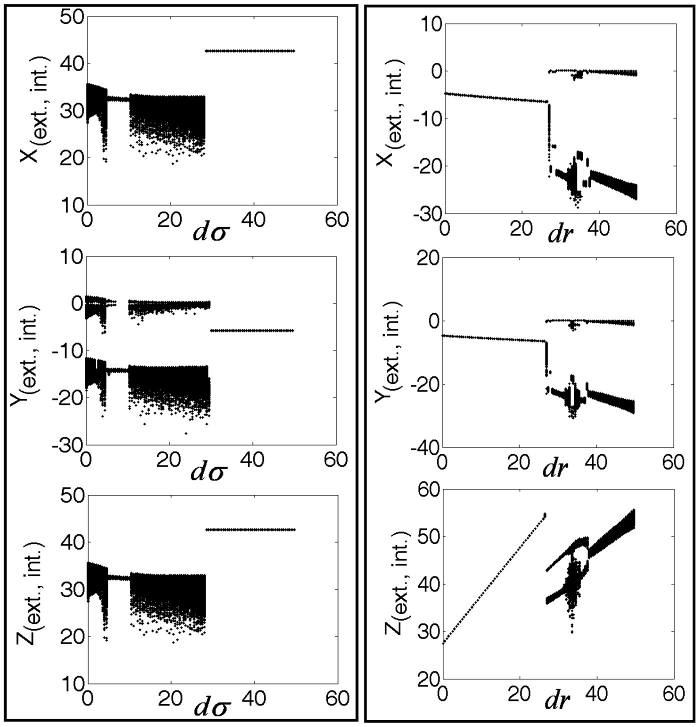

Figure 9. (a) Bifurcation plot showing the sensitivity of the supply chain to the internal variable $d\sigma$ (b) Bifurcation plot showing the sensitivity of the supply chain to the internal variable dr

perturbed supply chain system was very sensitive to tiny/small variations of the internal parameters of the supply chain. Indeed, it was observed various states of the supply chain raging from regular to chaotic states. These states were observed when monitoring the internal parameters (e.g. $0 \le d\sigma \le 50$ and $0 \le dr \le 50$) of the supply chain. Therefore, the bifurcation analysis can help to discover the various states towards the achievement of synchronization. This analysis can also be used to control and cancel the effects due to external perturbations.

Figures 9(a)-9(b) are bifurcation diagrams showing the states of the perturbed supply chain. The control parameters $d\sigma$ and dr are obtained by adjusting the internal parameter σ and r respectively. Figure 9 show the extreme sensitivity of the supply chain to the variations of $d\sigma$ and dr. Indeed, windows of regular states are shown which alternate with windows of chaotic states (e.g. period-1, period-3, and chaotic attractors are shown). From Figure 9, windows of parameters can be defined in which each of these states can occur. A control or cancellation of these states is possible and can be achieved through the synchronization analysis.

Bifurcation diagrams are of necessary importance as they can be used to define the ranges of the internal parameters of the supply chain in which the synchronization can be achieved. Two main conditions are important for the achievement of synchronization. The first condition is related to the periodicity. The second condition for the achievement of synchronization is described in Eq. (7). For instance, considering the periodicity of the original supply chain i.e. period-1 attractor (Figure 3), the synchronization (or the annihilation/cancellation of the effects due to external perturbations) to be achieved must lead to the same periodicity. This is the result/consequence of the competition between the internal variation of the system values and the effects of external perturbations. It can be derived from the first condition related to the periodicity the following windows $5 \le d\sigma \le 10$, $30 \le d\sigma \le 50$ and $0 \le dr \le 27$ (Figures. 9-10) in which the synchronization of the supply chain can be achieved. These windows define the set of the internal parameters settings in which the computation must be performed to fulfill the second condition (Eq. (7)). It should be worth noticing that the bifurcation diagrams were exploited to define the ranges or windows of the internal parameters of the supply chain in which the regulation process can be performed. A random choice of these windows to perform synchronization is possible as well. Nevertheless, when computing in random windows it is not possible to know if the achievement of synchronization is possible. Therefore, the method based on the bifurcation diagram is a systematic tool that can be exploited by strategic decision makers to evaluate how far their supply chain can be affected if the parameters settings are changed.

VI. Tamagotchi™ supply chain

Tamagotchi™ is the first simulation game of the virtual pet class. In 1996, Bandai & Co. introduced Tamagotchi products in to the toy market. Bandai is a famous product vendor for the popular characters, such as POWER RANGERS, GUNDAM, and DIGIMON. Bandai divided its products into eight divisions such as character goods for boys, vending machine products, video games and general toys, models, toys for girls, apparel, snacks and others.

Tamagotchi™ appears like an egg-shaped computer game and categorized in the video games and general toys division. The basic idea of Bandai to introduce the Tamagotchi™ is to create a soft pet toy, the way to play with Tamagotchi™ is to take care of it by feeding, giving an injection and so on. Bandai forecasted well in advance at the conceptual stage about the potentiality to strike the toys market even without advertisements using mass media. They initially forecasted that the sales will hit by 300 thousand units by the end of 1996 only in the domestic market. However due to spread of mouth publicity and the immense interest for multiple number of toys Bandai sold about 450 thousand by the end of the 1996 and 4 million by the end of the march, 1997 [31]. However, Bandai was unable to satisfy the customer demand with its supply chain getting affected by the shortage game, copy problems creating phantom demand. In addition, Bandai received more complaints every day about the shortages through wide communication sources and received reports to the police man concerning robberies and aggravated assaults to acquire

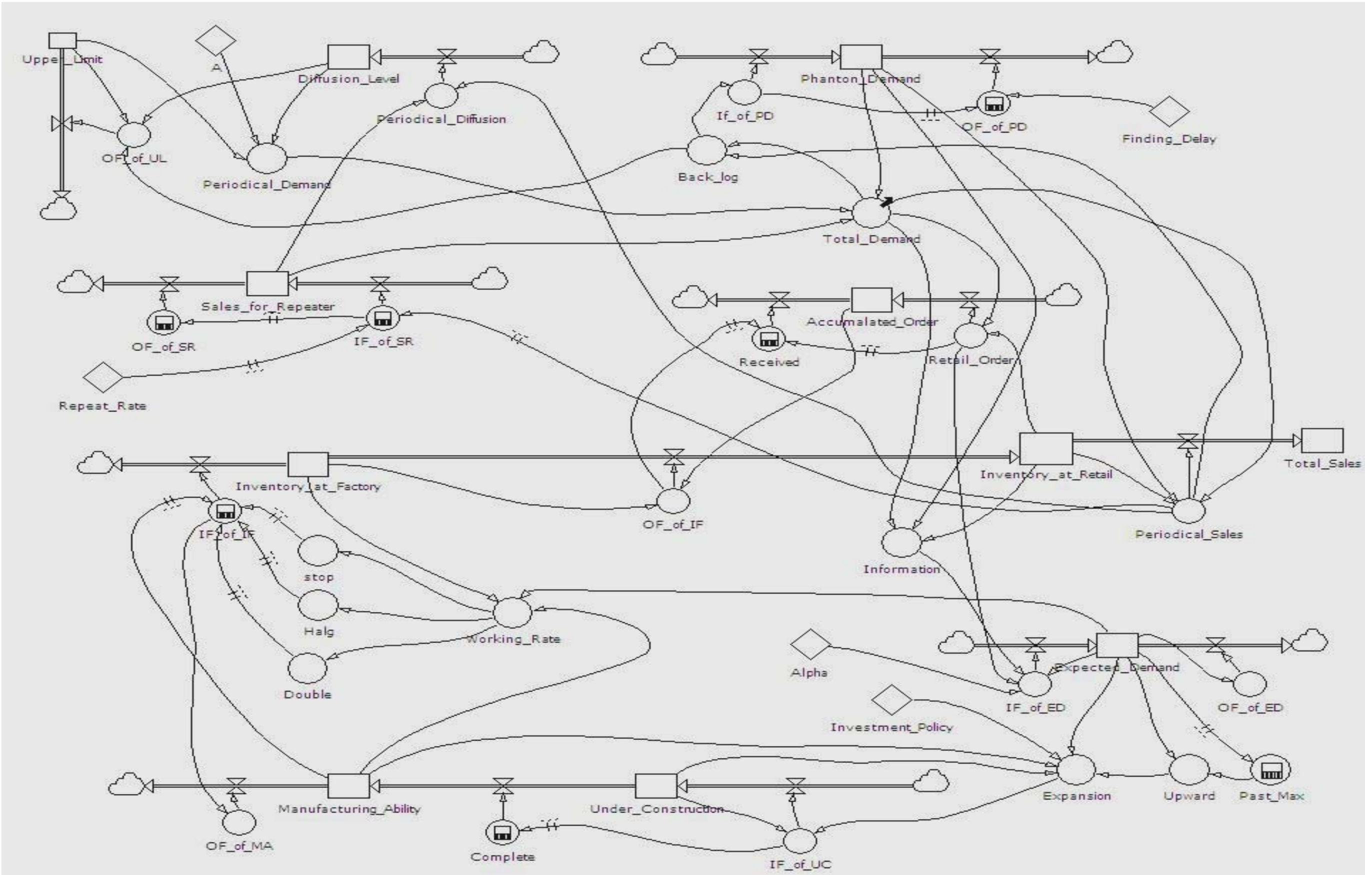

Figure 10. State flow diagram of Tamagotchi™ Supply chain model

Tamagotchi™ toys. Finally, Bandai understood the consequences while maintaining overstock and excess capacity and instantly expanded their manufacturing facilities. After expansion of manufacturing facilities Bandai encountered a sharp decline of demand and loses 16 billion yen in fiscal year 1998 [31]. The analysis shows that Bandai was too influenced by the boom and the bullwhip effects.

The above case study illustrates the influence of bullwhip effect and stock out problems. To understand and demonstrate the prevention of these tremendously unfortunate effects Higuchi [31] proposed a system dynamics based approach to conduct sensitivity analysis. To analyze the complex systems Forrester introduced Systems Dynamics concepts [19]. Systems Dynamics is a well-elaborated methodology for deterministic simulation and also analyzes the movements of dynamic systems.

Tamagotchi supply chain model considers three levels, the manufacturer, retail and customer market as described in our theoretical there level model of the supply chain. The scenarios for the market, retail and the factory are described using conceptual framework and then demonstrated the effect of shortages. At market level, the total demand value is calculated as the sum of demands for new customers, phantom demands and sales for repeaters. In addition, the diffusion speed of new products into the market is expressed by using logistic curve i.e. an S-Shaped curve with α and 15% population as initial upper limit. At the retail and factory levels, demands are reviewed and forecasted every week. The issue is here to identify forecasting method that best fits in this case study. Qualitative such as subjective curve fitting, the Delphi method and quantitative methods where experts play a vital role to analyze the next are used to forecast the demand value. During last-period moving average and exponential smoothing are useful methods to invalidate the variations. However, the above statements explain the main advantage of exponential smoothing and significance than the moving average method.

To model the conceptual framework of the Tamagotchi™ case study following assumptions are made: the review of production volume is every week and manufacturing delay is three weeks. The initial manufacturing capability is expected to be 37,500 units per week. The initial maximum manufacturing capability was assumed to be 75,000 units per week by over time and other methods, actual sales is 45,000 units per week in the first six weeks.

The system dynamics based simulation model as shown in Figure 10 is modeled in powersim® to demonstrate the dynamics of the supply chain. Bifurcation analysis is performed on the key parameters like diffusion speed and rate of repeat for sale for the product with respective to inventory at factory as shown in Figures 11-12. This analysis helps to understand for which parameter range the inventory at the factory can be in steady state. This bifurcation analysis gives the strategists in understanding the sensitive parameter range better as explained in section V.

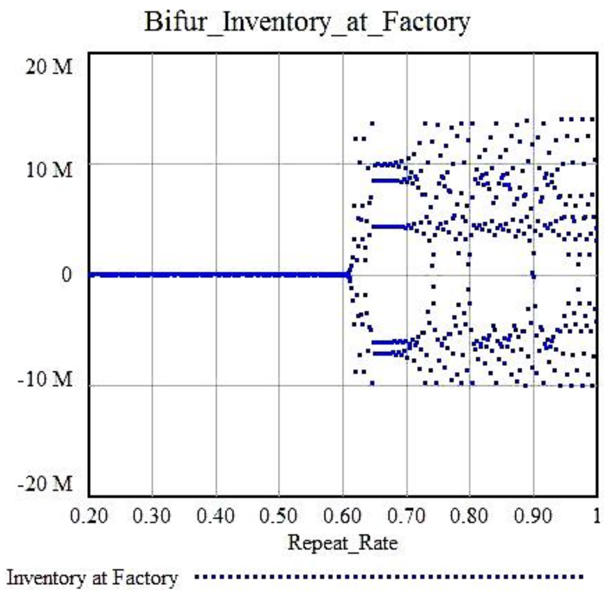

Figure 12. Bifurcation plot showing the sensitivity of the supply chain to the repeat rate

VII. Conclusions

The management of supply chains is a complex issue which involves numerous time varying dynamic situations. This work was concerned with the investigation of the effects of uncertainties generated by the dynamic and volatile global market-place on the stability of a three level supply chain. The dynamics of this supply chain was modeled mathematically by the well-known Lorenz oscillator. The uncertainties were considered as periodic external perturbations. The mathematical models were derived by exploiting the structure of the three level supply chain proposed in this work. We have illustrated how a supply chain can exhibit chaotic modes causing forester effect and also saturation modes when subjected to external perturbations. These two modes show the states of instability of the three level supply chain subjected to perturbations. A regulation scheme was designed and exploited to cancel or alleviate the effects due to external perturbations. It was shown that this cancellation or alleviation leads to the achievement of synchronization which is characterized by the reestablishment of the reference data in the supply chain. Two main criteria were defined for the achievement of synchronization. These criteria were exploited to derive some appropriated values of the system parameters for the achievement of synchronization. The regulation process was based on the variation of the internal parameters of the supply chain in well specified ranges/windows. These are ranges/windows within which the achievement of synchronization is possible. The challenging issue was based on the method to derive or determine these ranges/windows of parameters. We have shown that the bifurcation analysis was appropriate to determine these ranges/windows. The bifurcation analysis was carried out where two important internal parameters (i.e., rate of customer demand satisfaction and rate of information distortion at distributor) were considered as control parameters. Some bifurcation diagrams were obtained showing the extreme sensitivity of the three echelon supply chain when subjected to both external and internal perturbations. It has been found through bifurcations diagrams that the effects due to perturbations can lead to both chaotic and regular states of the supply chain and that these states alternate when monitoring the internal parameters of the supply chain. The bifurcation analysis in this work has been shown to be of necessary importance as it could help the strategic level decision makers in better understanding of the performance of the supply chain over a range of parameter settings. The regulation process exploited in this work was based on an adaptive algorithm for the automatic cancellation of the effects of the external perturbations by re-adjusting the internal thresholds. This process is particularly appealing as it is possible to control or adjust the internal thresholds of the supply chain. The solutions proposed in this paper offer a new range of possibilities for risk managers and provide a future research direction with the aim of considering the concept of nonlinear dynamics.

An open question under investigation concerns the design of “analogue computing” based simulators based on the CNN (Cellular Neural Network) technology to achieve the adaptive synchronization in supply chain networks. This investigation is of high importance due to both the complexity and dynamic character of supply chain networks in practice. These features make the supply chains very difficult to simulate by mean of the classical simulation tools. It would also be of great interest considering the case where the external perturbations to which the supply chain networks are subjected are non-periodic and

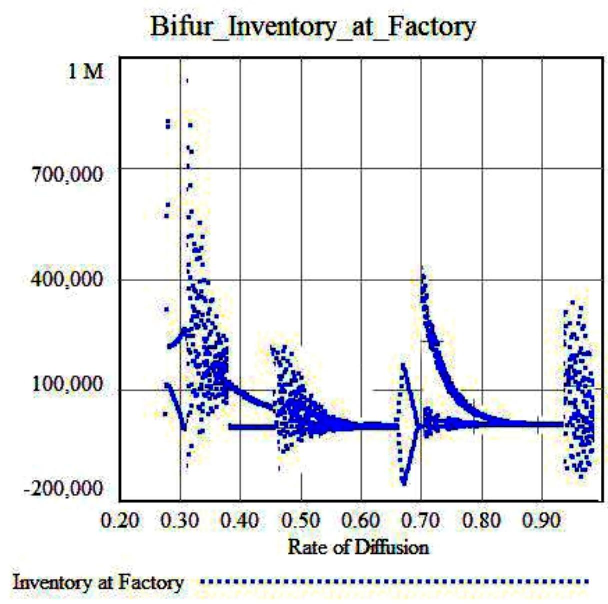

Figure 11. Bifurcation plot showing the sensitivity of the supply chain to the rate of the diffusion

stochastic. This is a realistic scenario which currently manifests itself in commercial supply chain networks and which can reflect the evolution of the market demand.

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