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Bifurcation analysis and synchronisation issues in a three-echelon supply chain

In today's global market-place, supply chains are dynamic and volatile. This dynamic and volatile nature produces various types of uncertainties along the supply chain, for example demand uncertainty, supply uncertainty, delivery uncertainty and forecasting uncertainty. These uncertainties make supply chains complex and non-linear systems as they propagate along the supply chain in both upstream and down stream. This work investigates the dynamic behaviour of a three-echelon supply chain. The modeling of this structure is carried out to display its non-linear dynamic behaviour. It is shown that the dynamics (stability) of the supply chain is very sensitive to external uncertainties. Specifically, the supply chain subjected to these uncertainties can exhibit strange and undesired states such as saturation and chaos. An adaptive algorithm for the automatic cancellation of these strange dynamics due to uncertainties is developed by re-adjusting the internal parameters of the supply chain in order to achieve its synchronisation. A bifurcation analysis is carried out. This analysis is essential and useful for strategic decision makers as it allows both the visualisation and control of the states/dynamics of the entire supply chain. The internal parameters of the supply chain are used as control parameters, and various remarkable states are discovered towards the achievement of synchronisation.

Keywords*:* Supply chain synchronisation; Supply chain control; Bullwhip effect; Chaos in supply chains; Bifurcation analysis in supply chains.

Introduction

Supply chain network management has been defined as the management of upstream and downstream relationships with suppliers and customers in order to create enhanced value in the final market-place at less cost to the supply chain as a whole (Jüttne et al., 2007). Today's global market-place is increasingly dynamic and volatile. This dynamic and volatile nature produces various types of uncertainties along the supply chain, for example, demand uncertainty, supply uncertainty, delivery uncertainty, and forecasting uncertainty. Apart from these uncertainties caused by external sources, uncertainties observed on a daily basis (for example machine breakdowns, wrong supplies and supply shortages.) make supply chains much more complex systems. The uncertainties propagate along the upstream and downstream of the entire supply chain leading to the production of various non-linear dynamic effects. In addition to these uncertainties, the relationship between the various players in the supply chain is often characterised by mistrust and competition.

Moreover, inventory is generally used as insurance against the uncertainties. In the case of a single enterprise-based supply chain it is relatively easy to overcome the uncertainties with properly sized inventories at each stage such as raw materials, work in process and finished goods inventories. The present day statistical tools and forecasting methods can satisfactorily aid in determining how much must be held to satisfy the customer demand for the particular product despite the uncertainties (Davis, 1993). However, the problem is much more complicated when considering the whole network consisting of different players distributed globally. Practically each player holds some inventory to protect against uncertainties, but the real difficulty is in determining how much must be held and where to hold it. To date, there is no clear analytical way to calculate the propagation of uncertainties up and/or down the supply chain. Traditionally, firms have relied on the experience and intuition in facing uncertainties.

The decisions made with intuition can make the supply chain exhibit various chaotic states. Chaos is defined as an aperiodic, unpredictable and bounded dynamic in a deterministic system with sensitivity dependence on initial conditions (Crawford, 1991). Chaos is a disorderly long-term evolution occurring in deterministic non-linear systems. The beer game developed at MIT to introduce students and industrialists to the concepts of economic dynamics shed further light on supply chain dynamics (Stermann, 1989). It has been found that one in four management teams in the supply chain creates deterministic chaos in the ordering pattern and inventory levels (Wilding, 1998). This clearly demonstrated in practice the occurrence of chaos in supply chains.

The objectives of this paper are threefold: (1) to show the potential of the concept of non-linear dynamics and modeling in supply chain networks; (2) to show the interest of synchronisation in supply chain networks; and (3) to offer to strategic decision makers an approach or technique to control and stabilise the states of their supply chain networks when subjected to uncertainties.

Responding to uncertainty

The objective of any player in a supply chain network is to achieve maximum profits and give maximum customer satisfaction. In addition to the stated individual objective all players also have a responsibility towards the global objective of resilient supply chain networks (SCN).
Christopher (Christopher, 1992) emphasises this by stating:

> Competition in the future will not be between individual organizations but between competing supply chains

This realisation has made the entities in SCN to look beyond their own boundaries to assess how the resources of each other can be utilised to achieve the global objective without compromising on their own objective. In the midst of pursuing towards it, entities have relentlessly restructured and reengineered their internal organisational boundaries and policies with an objective of transforming their relations from “arm’s – length” relationship to “durable arm’s – length” relationships (Skjoett-Larsen et al., 2003).

The definition of a truly efficient supply chain is when all players involved are communicating correct data – a manufacturer is communicating the correct product information and receiving accurate purchase orders; a retailer is receiving the specific products that were ordered; and the product is available to the end consumer at the right time and at the right price. However, communicating the correct data is not always possible in supply chains as each stakeholder has different objectives and constraints. The traditional supply chain management has been based on limited information sharing restricted to the product in consideration and transaction-oriented towards that product (Ayers, 2006). It is well studied (Chen et al., 2000, Davis, 1993, Gunasekaran and Ngai, 2004, Hwarng and Xie, 2006) that information sharing, demand patterns, ordering policy, and lead time have a direct impact on the performance of supply chains. Information sharing can reduce the lead time. Lead time reduction is found to be very beneficial and can reduce inventory and demand variability and improve customer service (Chen et al., 2000).

However, during the process of identifying the ways to mitigate the effects due to uncertainties, companies within the supply chain realised that they need to achieve the self organisation of the supply chain they belong to in order to satisfy the stated objective. In accordance with the greater focus on the self-organisation of the supply chain, the companies are increasingly focusing on the pre-requisities such as integration, collaboration and synchronisation between all entities in the supply chain as shown in Figure 1.

The first step towards self-organised supply chains is the integration stage. A Complex corporate structure demands the various functional units within a company to be integrated first. The intra-entity integration also called the functional integration is the basic driver towards the integration of the entire supply chain. The functional integration of purchasing, manufacturing, transportation and warehousing activities gives the very much needed visibility to the supply chain. Besides the functional integration, the inter-temporal integration also called hierarchical integration of these activities over strategic, tactical, and operational levels is also important (Anne et al., 2008a, Ayers, April 2006, Jespersen and Skjott-Larsen, 2005). This inter-temporal integration requires consistency among overlapping supply chain decisions at various levels of planning. However, the major role of the inter-temporal integration is in designing the supply chain for the product. The improved integration of activities across multiple companies/entities in a supply chain allows the collection of data from the breadth of the supply chain (Fawcett and Magnan, April 2002). Information technology is the key enabler for integration in supply chains. The information systems such as EDI (Electronic Data Interchange) and ERP (Enterprise Resource Planning) provide the necessary information needed for the integration (Lyons et al., 2005). The information systems make the information available; however the effect of integration directly depends on the quality of the information made available.

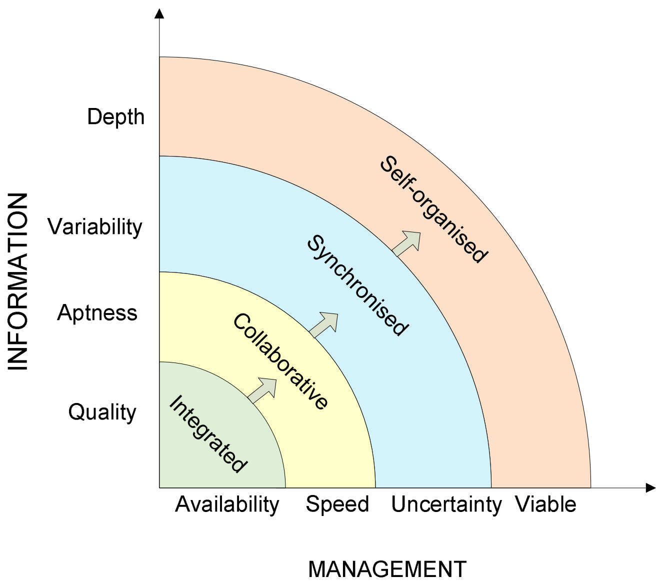

Figure 1. Key stages to the evolution of adaptive supply chain networks

After the integration of the supply chain, the next step towards self-organisation is the collaboration between entities. The objective of the collaboration is to make the information available when it is needed. Of course collaboration concepts are not new; they exist from the days of traditional business in the form of contracts. However, the effectiveness and execution speed, of the collaborations is highly increased due to the technological advances and the integration tools. Collaboration is strong where business to business relationships are strong. The degree of collaboration varies depending upon the strength of the integration (Holweg et al., April 2005). However, true collaboration among and in between all the entities in a SCN is more difficult in practice.

Even though data is made available with the help of integration tools and collaborative agreements, often the collected data paints a false image of an operation due to data entry errors and inconsistent collection procedures. Further, the upstream/downstream requirements are sometimes not clearly understood, not accurate enough and are not up-to-date. The breadth of the supply chain can compound the accuracy problem as the data can be re-worked or re-created in between.

In order to cope with inaccurate and/or varied data, synchronisation is an important step in dealing with uncertainties. Synchronisation can be classified into two types, namely the complete synchronisation and the partial synchronisation. The complete synchronisation is observed when the data synchronisation is achieved, leading to the real-time access to available data by all players at the same time. Complete synchronisation enables the supply chain to react quickly to changes in demand and in product design. This type of synchronisation is particularly suitable in just-in-time supply chain networks (Mondragon and Lyons, 2007). To achieve the complete synchronisation the complete chain should follow the integration and collaboration methods in true spirit.

Partial synchronisation is achieved through a feedback controller item. Apart from the data synchronisation as explained in complete synchronisation, a controller item is developed to mitigate the effects due to both inaccurate data and uncertainties. In this type of synchronisation the major effort is placed in quantifying the effects due to uncertainties. The modelling and quantification of the effects caused by the time lag (i.e. time delay), the information discrepancy, and the individual objectives help in designing the controller/synchroniser element. This controller item can be unique for each entity or supply chain as a whole. Uncertainties and exceptions are identified early and the data for intelligent response are immediately available. This greatly minimizes the bullwhip effect, demand amplification, and saves downstream partners and customers from needless activity (Anne et al., 2008a, Anne et al., 2008b, Hwarng and Xie, 2006, Lei et al., 2006). In this paper, we propose an adaptive controller to achieve synchronisation phenomena in order to mitigate the effects due to uncertainties.

Supply chain modeling

The theoretical framework for supply chain management underlies the setting, optimisation and control of the system model. The system model is not unique for all the supply chains (Poirier, August 2003). The system dynamics change for each type of product, for example,. food, oil, consumer goods, etc., depending upon the processes involved. The system dynamics based approach to model the business dynamics was first introduced by Forrester (Forrester, 1961). System dynamics has its origins in control engineering and management. The approach uses a perspective based on information feedback and delays to understand the dynamic behaviour of complex physical and social systems. System dynamics is an approach which is actively used to model the managerial behaviour.

In 1989, Sterman (Stermann, 1989) proposed a model that can be used to analyse the supply chains using the industrial dynamics fundamentals proposed by Forrester. The Sterman model was actively used to analyse the supply chain dynamics. Due to its visual nature and simplicity, after a lengthy floppy period, the system dynamics approach is gaining momentum in modelling the inventory management processes, in policy development and demand amplification (Angerhofer and Angelides, 2000). The system theory based modelling is also used to develop the feedback controllers to mitigate the bullwhip effect (i.e. the demand amplification) to some extent (Laurikkala et al., 2003). However, to address the issues emanating from uncertainties and the problems occurring in real time, system dynamics modelling might not be suitable (Laurikkala et al., 2003, Chen et al., 2000, Angerhofer and Angelides, 2000).

In recent years, many researchers are also using an agent-based distributed modelling approach to cope with supply chain networks (SCN) (Nissen, 2001, Stadtler and Kilger, 2000, Jespersen and Skjott-Larsen, 2005). One or several agents can be used to represent each entity in the SCN. Each agent is assigned with both a local objective and global objective. With the advent of mobile agents which can run on lighter platforms, the use of agents to both collect information and take decisions has become popular. Moreover, the agent paradigm is a natural metaphor for network organisations, since companies prefer maximising their own profit rather than the profit of the supply chain (Nissen, 2001). The multi-agent based modelling approach offers a way to elaborate the supply chain as the agents are autonomous and the agent rules can be defined in advance. The distributed decision nature of the multi-agent systems makes it easier to add other entities in the local environment. Entities leaving the SCN in the middle will not affect the entire SCN to a great extent. The agent rule framework provides a certain amount of trust among the partners as it eliminates the mistrust and deception among entities.

The major disadvantages with the multi-agent based SCN modeling is that, there is no global view of the system theoretical optimisation, the optimisation of the supply chain cannot be visualised. As per the system dynamics theory any system can be unstable (Forrester, 1961); this theorem implies that the multi-agent system, which is a system with multi-autonomous elements/entities, again can be unstable. Further, the multi agent system is also based on the assumption that all the participating entities in the SCN are truly integrated and collaborating. However "true integration and collaboration" is highly difficult (Lei et al., 2006, Dejonckheere et al., June 2003, Stermann, 1989 , Akintoye, December 2000, Khouja, 2003, Hwarng and Xie, 2006).

By considering these issues (in this work) we envisage the complex non-linear modelling of a three-echelon supply chain to represent the realistic dynamics. A three level model is envisaged (Figure 2) to describe a simple scenario in a very complex supply chain. The nonlinear supply chain models in the literature (Giacomo and Patrizi, 2006, Poirier, August 2003) focus mainly on the specific tasks, and thus becomes a transaction-oriented approach. In this work, we mainly focus on building a non-linear supply chain model that can exhibit more complexity covering the information distortion, retailer order satisfaction and safety stock. An additional criterion is the extreme sensitivity of the model to both uncertainties and initial conditions. The following notations are introduced to facilitate the description of the model:

i	Time period
m	Rate of customer demand satisfaction at retailer
r	Rate of information distortion of products demanded by retailer
k	Safety stock coefficient at manufacturer
x_i	The quantity demanded by retailer in current period
y_i	The quantity distributors can supply in current period
z_i	The quantity produced in current period depend on the order

The orders they make might not be equal to orders they receive. The order-out quantity depends not only on how much inventory you have already, but also how much you want to supply out. The order-out quantity at the retailer depends on the ratio m at which the demand is satisfied during the previous order. The distributor needs to take into consideration among other things, the rate of information distortion r that can occur in the received orders. The producer needs to note the safety stock k in order to avoid the small production batches. These scenarios/phenomena are described in Figure 2. An in-depth explanation is provided below and a corresponding mathematical model is derived to analyse the dynamics of the SCN.

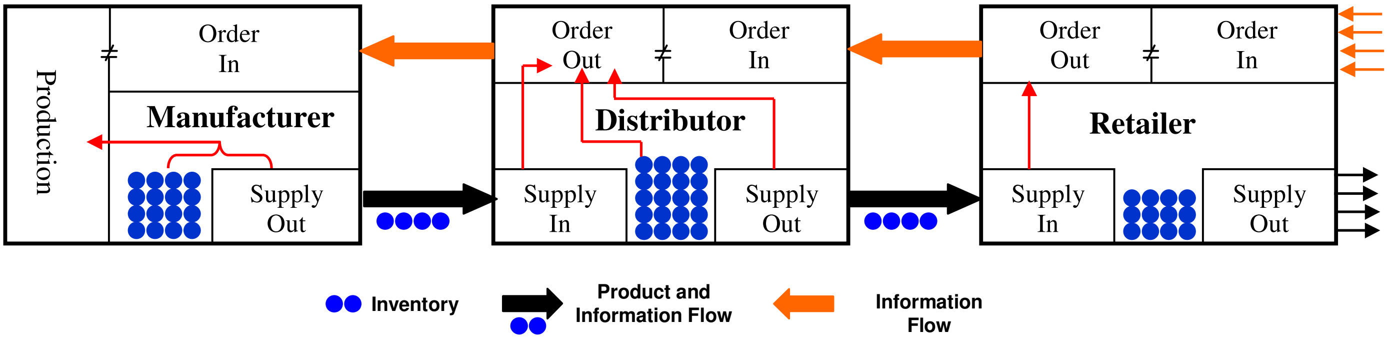

Figure 2. Model of a three echelon supply chain

We consider that the demand information is transmitted within the layers of the supply chain with a delay of one unit of time. As illustrated in Figure 1, the ordering quantity is not the same as the requested order quantity at any level. The order quantity at the current period of time at the retailer is linearly coupled with the distributor and it is influenced by how much demand is satisfied in the previous period of time. This scenario/phenomenon is modelled by Eq. (1).

$$x_i = m(y_{i-1} - x_{i-1}) \qquad (1)$$

Here m is the ratio at which the demand is satisfied. The dependency/coupling between the distributor, the producer and the retailer (Figure 1) is not linear. Indeed the distributor needs to take the combined effect of the retailer and producer into consideration before making his order, i.e., quadratic coupling. Apart from this, the distributor also needs to take into consideration that the order information received from the retailer might be distorted. This scenario is modelled by Eq. (2).

$$y_i = x_{i-1}(r - z_{i-1}) \qquad (2)$$

Here, r is the information distortion coefficient. The production quantity from the producer unit typically depends on the distributor's orders and the safety stock. However the distributors' orders again depend on the retailer's orders, i.e., the producer needs to take the combined effect of retailer and distributor into account before making production decisions. This scenario is modelled by Eq. (3)

$$z_i = x_{i-1}y_{i-1} + kz_{i-1} \qquad (3)$$

Eqs. (1) – (3) represent the quantity demanded by customers (Eq. (1)), the inventory level of distributors (Eq. (2)) and the quantity produced by producers (Eq. (3)),
where:

$x_i < 0$ denotes that the supply is less than customers demand in the previous period

y_i <0 denotes that the information is severely distorted and no adjustment is necessary at the inventory level
z_i <0 denotes the cases of overstock or return and hence no new productions

Eqs. (1) - (3) are discrete models describing the dynamics of the SCN of Figure 2. Considering very small time intervals, the continuous model in Eq. (4) can be derived from Eqs. (1) - (3).

$$\begin{cases} \dot{x} = my - (m+1)x \\ \dot{y} = rx - y - xz \\ \dot{z} = xy + (k-1)z \end{cases} \qquad (4)$$

If the conditions $\sigma = m+1$ and $b = 1-k$ are satisfied, Eq. 4 leads to the *Lorenz* model in Eq. 5.

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = rx - y - xz \\ \dot{z} = xy - bz \end{cases} \qquad (5)$$

From the theory of dynamic systems it is proved that this model produces a wide variety of non-linear features depending upon the parameters' values. This model is particularly of interest when dealing with the modelling of scenarios/phenomena which are very sensitive to initial conditions and to uncertainties as well. In the case of SCN, uncertainties, when added at one layer effectively propagate in both upstream and down stream. This is common dynamics exhibited by realistic SCNs. A similar model (Eq. (5)) is also proposed by the authors (Lei et al., 2006) to exhibit the supply chain dynamics and mitigate the bullwhip effect. In this work, we are considering the external uncertainties caused by external perturbations. These perturbations can occur due to the market-place dynamics and volatility. We consider the external perturbations to be non-linear as the market-place behaviour is non-linear in nature. Assuming that the perturbations can affect any of the three levels of the SCN, a perturbed form of Eq. (5) is proposed in Eq. (6).

$$\begin{cases} \dot{x}' = \sigma(y' - x') + d_1 \\ \dot{y}' = rx' - y' - x'z' + d_2 \\ \dot{z}' = x'y' - bz' + d_3 \end{cases} \qquad (6)$$

Where d_i $(i = 1,2,3)$ represent the external perturbations.

Before considering the effects of external perturbations, let us focus first on discussing the phase portrait of a given supply chain for illustration. For this illustration we define an example supply chain with the following parameter values for the model in Eq (5): $\sigma = 15$ $r = 29$ and $b = 2/3$. Before perturbation we call this system example "reference supply chain model".
The "phase portrait/diagram" is the set of all possible states of a dynamical system; each state of the system corresponds to a unique point in the phase space (Terman and Izhikevich, 2008). considering two system variables, X and Y, we can plot one variable against the other at a given point in time on a standard XY graph. This is called a "phase portrait/diagram" of the system. The form and structure of the phase portrait reveals information about the system behaviour for the chosen parameter values: in case of the form of a point, the

system is in saturation; in case of a periodic orbit, the system is in regular state; and in case of discernible patterns, the system is in a chaotic state (Terman and Izhikevich, 2008). Figure 3 displays the phase portrait of the example supply chain system, whereby X_{ref}, Y_{ref}, Z_{ref} represent the x, y and z of the three-level supply chain described in Eq. (5); its form reveals that the system is in a regular state.

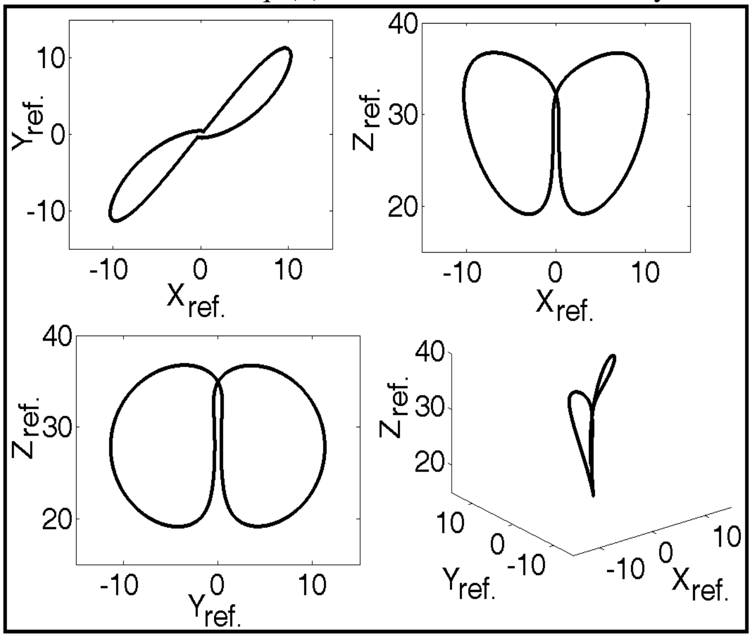

Figure 3. Phase space representation of the reference supply chain for $\sigma = 15$ $r = 29$ and $b = 2/3$

After defining the reference model we analyse the effects of external perturbations on this model. Basically we are concerned with the new dynamics exhibited by the reference supply chain subjected to external perturbations.

Synchronisation of an externally perturbed supply chain

In this section, we briefly discuss the traditional approach to investigate synchronisation issues. Modern synchronisation tools provide an automation framework but do not concentrate on what happens if the given data is slightly changed accidentally. Many companies have taken inspiration from the modern web and wireless technologies to facilitate synchronisation in a timely manner (Anne et al., 2008a, Khouja, 2003). The integration efforts and the collaboration for the processes within the SCN certainly improved communication by means of EDI and current internet-based, information exchanges. Better information (point of sale data and the Collaborative Planning, Forecasting, and Replenishment, CPFR, initiatives), and a general willingness to work more closely together made the timeliness of information possible to a certain extent. Nevertheless, the efficiencies have been gained through improvements that any executive can effect at his or her own workplace by putting in place the appropriate company-wide initiatives aimed at improving the internal business process. However, as we have seen, the uncertainties propagate in both directions (upstream and downstream) along a SCN, so network-wide initiatives are necessary to mitigate the effects caused by uncertainties.

In this context, we provide different cases of perturbations affecting the data and present techniques/methods to synchronise or stabilise the new states (or perturbed states) exhibited by the SCN. The causes of instability of the supply chain can be broadly classified into two categories. The first cause is the dynamical and non-linear character of the motions (i.e. material/products flow and information exchanges) between different entities in supply chains. The second cause originates from the effects of both external and internal perturbations (Lei et al., 2006) to which the supply chain is subjected.

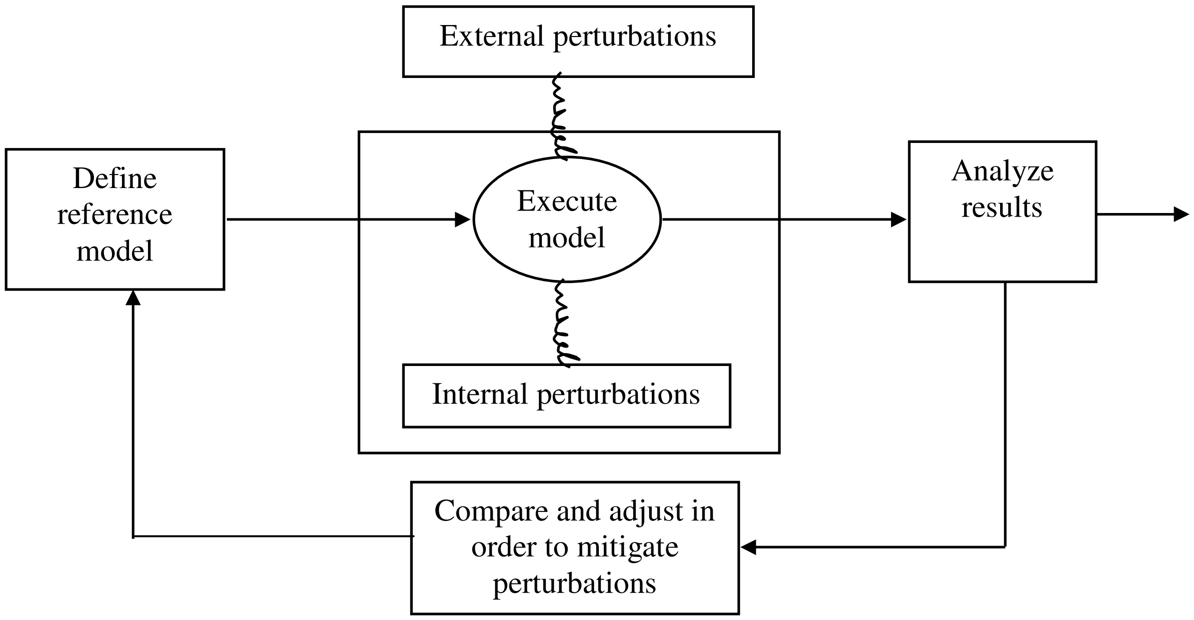

Figure 4. An adaptive feedback control model to mitigate the effects due to uncertainties and perturbations

An optimal management of the information flows within the supply chains may be of high importance in order to alleviate the effects leading to negative consequences on the flows within the supply chains. This could be achieved through an adaptive control mechanism which is based on a current comparison of the dynamical data within the supply chains with the pre-defined data fixed by the requirements of the supply chains. Here, an automatic or adaptive control of the flows within the supply chains should be able to detect changes in the flows and act accordingly/consequently (by undertaking a given action) in order to alleviate the undesirable effects and therefore stabilise the system behaviour that has been perturbed. The achievement of synchronisation is observed when the action undertaken has allowed the recovery of the original behaviour (eventually thresholds or reference requirements) of the supply chain. The schematic description of the adaptive synchronisation of a supply chain is illustrated in Figure 4.

This research develops an adaptive method (algorithms and/or tools) for the systematic and automatic control of the flows within the supply chains. In fact, due to the dynamic changes as discussed (in time domain), some pre-defined settings or requirements within the supply chains (thresholds such as safety stocks) may be varying accordingly as a consequence of these perturbations. It should be worth mentioning that a combination of the simultaneous effects of both internal and external perturbations may be responsible for the dynamic motion variations (for example flow of materials and information exchange) within the supply chain. This is a concrete and/or realistic scenario as the supply chains of many companies are currently exposed to both types of perturbations. However, the analysis in this work is restricted to the case where the reference model is subjected to external perturbations due to the fluctuation in the market demand.

Figure 5 shows the phase space structure of the reference supply chain subjected to the external perturbations with X(ext,int), Y(ext,int), Z(ext,int) representing $\dot{x}', \dot{y}'$ and $\dot{z}'$ of the supply chain described in Eq. (6). External perturbation $d_i (i = 1,2,3)$, $d_1 = 0.56\cos(3\mathrm{t})$, $d_2 = 20\cos(5\mathrm{t})$ and $d_3 = 50\cos(10\mathrm{t})$, is particularly considered as non-linear perturbation as the external uncertainties (for example, market fluctuation and forecasting demand) exhibit nonlinear dynamics. The effects of external perturbations on the original (or specific) requirements of the reference supply chain are clearly shown by the well-known chaotic Lorenz attractors exhibited by the reference SCN subjected to perturbations. The cancellation process of these effects is achieved by exploiting the synchronisation controller item shown in Figure 4. The synchronisation process concerns two different models of the supply chain: (a) the reference model (unperturbed model) described by Eq.(5), and (b) the externally perturbed model described in Eq. (6). Following the active control approach of Liao (Liao, 1998), for the purpose of synchronisation we vary the internal parameters of the perturbed supply chain. In order to vary the internal parameters, we define the state errors between the perturbed system and the reference system by Eq. (7).

$$e_x = x' - x;\; e_y = y' - y;\; e_z = z' - z; \qquad (7)$$

x', y' and *z'* are perturbed states and *x, y* and *z* are unperturbed states. The synchronization problem in this context can be equivalent to the problem of stabilising the system shown in Eq. (7). This is possible through a suitable choice of the internal variables of the perturbed system. Moreover, the adaptive control algorithm

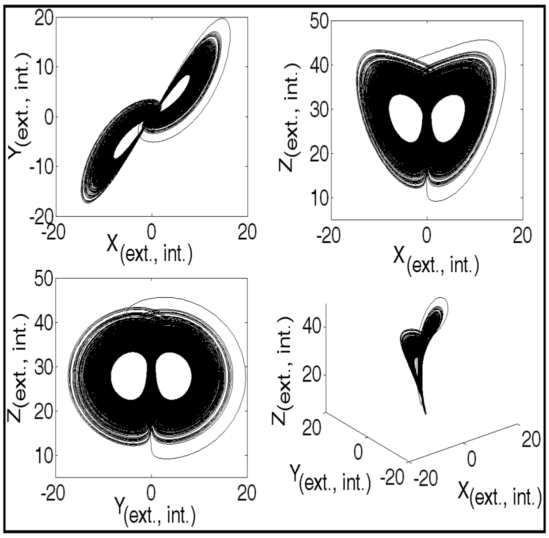

Figure 5. Chaotic state of the supply chain due to external perturbations

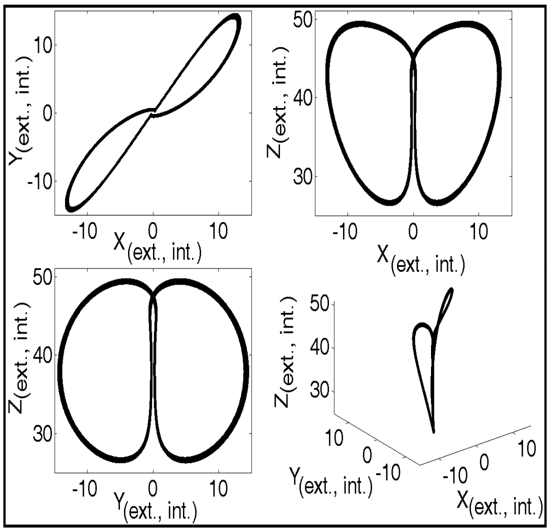

Figure 6. Alleviation of the chaotic effect caused by external perturbation with adaptive synchronisation

considers the effects of external perturbations and adjusts the values of the internal parameters (σ, r, b) of the three-echelon supply chain by tiny variations $d\sigma$, dr and db. The variation of each internal parameter is performed in well defined ranges (or windows) of variation. Performing the parameters variations in these ranges is necessary as we cannot vary the parameters beyond the realistic scenario. A threshold error is fixed (which is less than approximately 0.02) under which full alleviation of the effects due to external perturbations is supposed to be effective; this leads to the achievement of synchronisation, which results in the recovery of the behaviour of the reference system .

Figure 5 shows the structure in phase space of the supply chain subjected to external perturbations ($d_1 = 0.56\cos(3t)$, $d_2 = 20\cos(5t)$ and $d_3 = 50\cos(10t)$). This structure in the phase space shows the occurrence of the well-known bullwhip and chaotic effects in the three-level supply chain by displaying discernible patterns. The regulation process has been exploited to adjust the internal parameters values in order to achieve synchronisation, i.e. the full cancellation of the effects due to perturbations. The corresponding values obtained from the adaptive control processes as illustrated in Figure 4 for the achievement of the regulation process are $d\sigma = 35$, $dr = 15$ and $db = 0.09$. The results of this process are shown in Figure 6. Indeed, below the precision of 2% when compared with the reference model in Figure 3, Figure 6 shows attractors similar to those of the reference or original supply chain system.

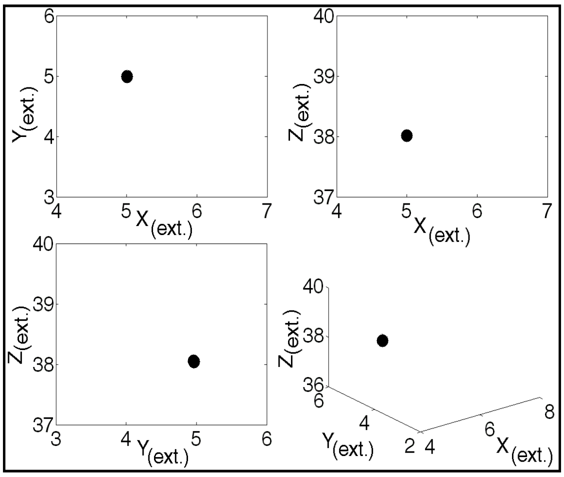

Figure 7. Saturation state of the supply chain due to external perturbations

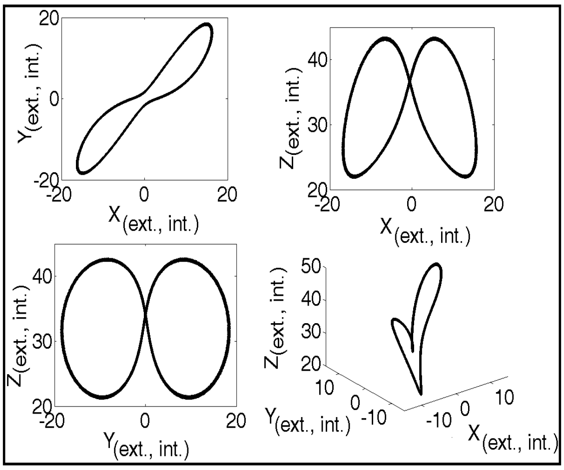

Figure 8. Alleviation of the saturation effect caused by external perturbation with adaptive synchronization

Further investigations have been performed to show that the supply chain subjected to external perturbations can exhibit the state of saturation which is characterised by non dynamic (or fixed) data in each level of the three levels. Indeed, for $d_1 = 10\cos(5t)$, $d_2 = 5\cos(10t)$ and $d_3 = 10\cos(10t)$, the achievement of the state of saturation is clearly shown in Figure 7. The saturation manifests itself by a sudden exhibition of fixed or constant values/data along each level of the externally perturbed three echelon supply chain. When the state of saturation is obtained, further changes/flows in the supply chain are not represented effectively. To alleviate the effect (i.e. saturation) due to external perturbations, we performed the adaptive regulation process explained before by adjusting the internal parameters of the SCN. The appropriate values of the internal parameters to alleviate the effects due to external perturbations are $d\sigma = 4$, $dr = 2$ and $db = 1$. The result of the regulation process is shown in Figure 8. Indeed, tiny variations of the internal parameters of the supply chain lead to the achievement of synchronisation. This is manifested by the abrupt change of the state of the system from the saturation state (Figure 7) to a regular state (Figure 8) which is similar to the state of the reference supply chain (Figure 3).

The supply chain subjected to external perturbations can exhibit various remarkable states such as chaos (Figure 5) and saturation (Figure 7) to name two. The regulation process can be performed to cancel or mitigate the effects due to external perturbations. This process is based on the adjustment of the internal parameters of the supply chain. Various new and interesting states of the supply chain are discovered towards the achievement of synchronisation which is characterised by the cancellation or alleviation of the effects due to external perturbations. Therefore, an interesting and open question must be concerned with the exploration of appropriate methods to control the states of the supply chain. Indeed, the stability of the supply chain is not robust (i.e. very sensitive) to external perturbations. The control process might lead to the derivation of the parameters ranges (windows) in which each of the various states of the supply chain can be found (or defined). The bifurcation analysis is an appropriate method to describe the various states of the supply chain in well specified windows of parameters.

Bifurcation analysis

The bifurcation is a qualitative change observed in the behaviour/state of a system as its parameters settings vary. The bifurcation is observed if the state of the system suddenly changes qualitatively upon small /smooth variation of the parameter values. The bifurcation theory (Crawford, 1991) is the analysis/study of the bifurcation scenarios with the aim of defining/determining the states (equilibrium/fixed points, periodic or chaotic states) of the system in a given parameter space. Basically, bifurcation values/points are critical values leading to qualitative changes in the states of the system.

The preceding section has shown that the annihilation or alleviation of the effects due to external

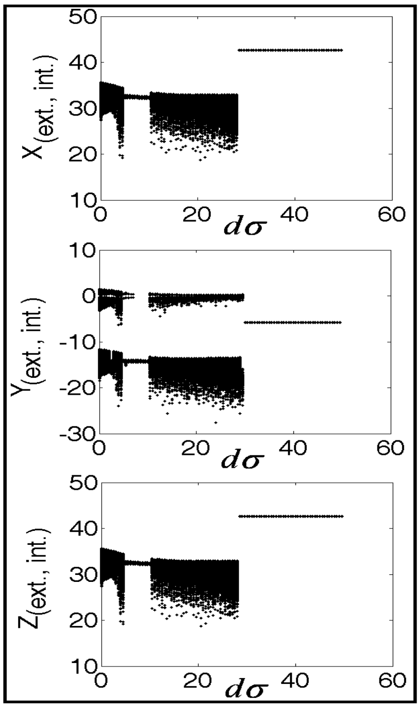

Figure 9. Bifurcation plot showing the sensitivity of the supply chain to the internal variable $d\sigma$

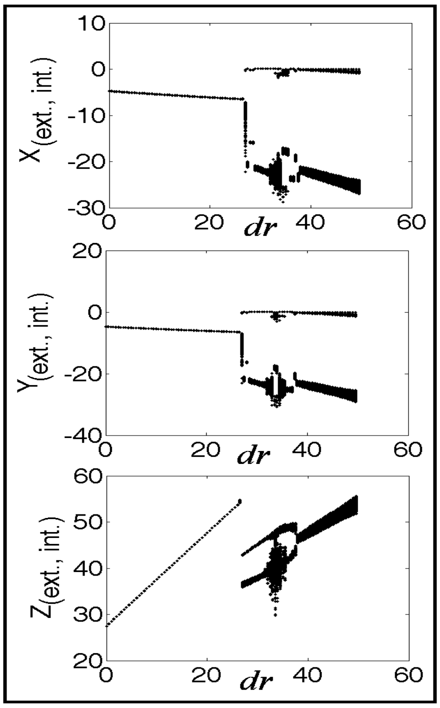

Figure 10. Bifurcation plot showing the sensitivity of the supply chain to the internal variable dr

perturbations is possible through the achievement of synchronisation. Nevertheless, during the regulation process, we found that the perturbed supply chain system was very sensitive to tiny/small variations of the internal parameters of the supply chain. Indeed, various states of the supply chain were observed raging from regular to chaotic states. These states were observed when monitoring the internal parameters (for example $0 \le d\sigma \le 50$ and $0 \le dr \le 50$) of the supply chain. Therefore, the bifurcation analysis can help to discover the various states towards the achievement of synchronisation. This analysis can also be used to control and cancel the effects due to external perturbations. Figures 9-10 are bifurcation diagrams showing the states of the perturbed supply chain. The control parameters $d\sigma$ and dr are obtained by adjusting the internal parameter σ and r respectively. Figures. 9-10 show the extreme sensitivity of the supply chain to the variations of $d\sigma$ and dr. Indeed, windows of regular states are shown which alternate with windows of chaotic states (for example period-1, period-3, and chaotic attractors are shown). From Figures. 9-10, windows of parameters can be defined in which each of these states can occur. A control or cancellation of these states is possible and can be achieved through the synchronisation analysis.

Bifurcation diagrams are of necessary importance as they can be used to define the ranges of the internal parameters of the supply chain in which the synchronisation can be achieved. Two main conditions are important for the achievement of synchronisation. The first condition is related to the periodicity. The second condition for the achievement of synchronisation is described in Eq. (7). For instance, considering the periodicity of the original supply chain i.e. period-1 attractor (Figure 3), the synchronisation (or the annihilation/cancellation of the effects due to external perturbations) to be achieved must lead to the same periodicity. This is the result/consequence of the competition between the internal variation of the system values and the effects of external perturbations. It can be derived from the first condition related to the periodicity in the following windows $5 \leq d\sigma \leq 10$, $30 \leq d\sigma \leq 50$ and $0 \leq dr \leq 27$ (Figures. 9-10) in which the synchronisation of the supply chain can be achieved. These windows define the set of the internal parameter settings in which the computation must be performed to fulfil the second condition (Eq. (7)). It should be worth noticing that the bifurcation diagrams were explored to define the ranges or windows of the internal parameters of the supply chain in which the regulation process can be performed. A random choice of these windows to perform synchronisation is also possible. Nevertheless, when computing in random windows it is not possible to know if the achievement of synchronization is possible. Therefore, the method based on the bifurcation diagram is a systematic tool that can be exploited by strategic decision makers to evaluate how far their supply chain can be affected if the parameters settings are changed.

Conclusions

The management of supply chains is a complex issue which involves numerous time varying dynamic situations. This work was concerned with the investigation of the effects of uncertainties generated by the dynamic and volatile global market-place on the stability of a three-level supply chain. The dynamics of this supply chain were modelled mathematically by the well-known Lorenz oscillator. The uncertainties were considered as periodic external perturbations. The mathematical models were derived by exploiting the structure of the three-level supply chain proposed in this work. We have illustrated how a supply chain can exhibit chaotic modes causing Forester effect and also saturation modes when subjected to external perturbations. These two modes show the states of instability of the three-level supply chain subjected to perturbations. A regulation scheme was designed and exploited to cancel or alleviate the effects due to external perturbations. It was shown that this cancellation or alleviation leads to the achievement of synchronisation which is characterised by the re-establishment of the reference data in the supply chain. Two main criteria were defined for the achievement of synchronisation. These criteria were exploited to derive some appropriate values of the system parameters for the achievement of synchronisation. The regulation process was based on the variation of the internal parameters of the supply chain in well-specified ranges. These are ranges within which the achievement of synchronisation is possible. The challenging issue was based on the method to derive or determine these ranges of parameters. We have shown that the bifurcation analysis was appropriate to determine these ranges. The bifurcation analysis was carried out where two important internal parameters (i.e., rate of customer demand satisfaction and rate of information distortion at distributor) were considered as control parameters. Some bifurcation diagrams were obtained showing the extreme sensitivity of the three-echelon supply chain when subjected to both external and internal perturbations. It has been found through bifurcation diagrams that the effects due to perturbations can lead to both chaotic and regular states of the supply chain and that these states alternate when monitoring the internal parameters of the supply chain. The bifurcation analysis in this work has been shown to be of necessary importance as it could help the strategic level decision makers to better understand the performance of the supply chain over a range of parameter settings. The regulation process exploited in this work was based on an adaptive algorithm for the automatic cancellation of the effects of the external perturbations by re-adjusting the internal thresholds. This process is particularly appealing as it is possible to control or adjust the internal thresholds of the supply chain. The solutions proposed in this paper offer a new range of possibilities for risk managers and provide a future research direction with the aim of considering the concept of non-linear dynamics.

An open question under investigation concerns the design of "analogue computing" based simulators based on the CNN (Cellular Neural Network) technology to achieve the adaptive synchronisation in supply chain networks. This investigation is of high importance due to both the complexity and dynamic character of supply chain networks in practice. These features make the supply chains very difficult to simulate by means of the classical simulation tools. It would also be of great interest considering the case where the external perturbations to which the supply chain networks are subjected are non-periodic and stochastic. This is a realistic scenario which currently manifests itself in commercial supply chain networks and which can reflect the evolution of the market demand.

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