

# A Novel Image Processing Approach Combining a ‘Coupled Nonlinear Oscillators’-based Paradigm with Cellular Neural Networks for Dynamic Robust Contrast Enhancement

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**Abstract**— In this paper, a systematic discussion of both pros and cons of two well-known traditional approaches for image contrast enhancement is conducted. The first approach is based on the CNN paradigm and the second one is based on the coupled nonlinear oscillators’ paradigm for image processing. In the later case an extensive bifurcation analysis is carried out and analytical formulas are derived to define the various states of the system. Both equilibrium and oscillatory states of the system are depicted. It is shown that each of these states has a significant impact on the quality of the resulting image contrast enhancement. A benchmarking is considered whereby a comparison is performed between the results obtained by a CNN-based processing, on one side, with those obtained by a ‘coupled non linear oscillators’ based processing, on the other side. The superiority of the later approach (for contrast enhancement) is demonstrated both analytically and through various experiments. A major drawback of the CNN based image processing is the practical inability to adjust/re-calculate templates in real-time in face of a dynamic scene with input images experiencing visibility and/or lighting related spatio-temporal dynamics. Finally, a novel hybrid approach integrating both schemes in an efficient way is proposed: the ‘coupled nonlinear oscillators’ based image processing is the main processing scheme that is however realized on top of a CNN processors’ framework. The hybrid approach does prove to overcome key practical problems faced by both original approaches.

**Keywords:** Cellular neural networks (CNN), Nonlinear coupled oscillators, van der Pol oscillator, Duffing oscillator, contrast enhancement, stability, bifurcation, Routh-Hurwitz theorem

## I. INTRODUCTION

The last decades have witnessed a tremendous attention devoted to the study of nonlinear coupled oscillators [2]-[17] with various related applications in diverse areas such as electrical engineering [18], [19], mechanics [15], electro-mechanics [14] and electronics [16], just to name a few. In some previous works [18]-[21], we have shown some interesting applications of the coupling between van der Pol and Duffing oscillators in both electronics and electro-mechanics. Further, in the recent literature a good number of notable contributions have been published thereby showing various applications of the paradigm of nonlinear dynamics in image processing [1]-[13]: (a) the use of the CNN paradigm for contrast enhancement [1], edge detection [11], [17], image segmentation [2]-[10], [12], [13]; and (b) the use of the so-called LEGION model (involving nonlinear coupled

oscillators) mainly for image segmentation [3]. One does realize that the relevant literature does not provide sufficient information concerning the application of nonlinear coupled/uncoupled oscillators in image processing, especially for the specific task of contrast enhancement. In fact, only a single paper can be found in which image contrast enhancement has been done by using this later paradigm [1].

In contrast, the cellular neural network paradigm has shown through numerous publications its rich potential to solve many important low-level image processing tasks, e.g. image contrast enhancement [22]-[24], edge detection [25] and segmentation [26], [27] just to name a few. Despite the ideal framework offered by the CNN paradigm to perform parallel and therefore ultrafast image processing there are still some important related issues that still need a better theoretical foundation. One of these open questions is that of a comprehensive and straight-forward methodology to derive appropriate CNN templates for a given image processing task. Actually known approaches are based on a sort of supervised learning paradigm to determine the templates. Hereby either genetic algorithms or simulated annealing or even particle swarm optimization are the most commonly used schemes. Thus, the template obtained through such a ‘supervised learning’ like approach does highly depend on the used reference image(s). Therefore, this traditional way of calculating templates will totally fail in face of a dynamic environment, which would require an adaptive and real-time determination/re-calculation of the respectively appropriate templates in reaction to visibility and lighting related environmental changes. Indeed, for a specific processing task (e.g. contrast enhancement, segmentation, etc.) the optimal CNN templates (for an optimal processing) must be adjusted / recalculated depending on the varying input image.

That’s why an important open key issue not yet answered so far by the relevant scientific community is that of developing/providing a comprehensive, robust and general framework that should allow a real-time adaptation of the CNN templates related to a specific image processing task to the variations in all aspects of the input image(s). It is known that CNN templates are very sensitive to the quality of the input image and must be adjusted in case of image dynamics for an optimal processing. The supervised learning template calculation paradigm is therefore not appropriate for a situation where the input image does experience visibility related temporal dynamics; it is almost impossible to recalculate the templates in real-time.

Thus, a key objective of this paper is to propose an approach or better an image processing (in this case for “contrast enhancement”) framework which is robust to both the temporal quality and the spatial changes of the input image(s). The novel approach proposed here does combine the paradigm of coupled nonlinear oscillators with that of cellular neural networks. It is shown how this integration should be realized at best. Afterwards, it is in the following steps clearly demonstrated that the new architectural framework does result in invariant templates while still being capable of robustly adapting the efficient image processing to the spatial-temporal dynamics of the input image.

The nonlinear coupled oscillator system model used in this paper does consist of the coupling between van der Pol and Duffing type oscillators. The focus is hereby on the application of this coupled system for the specific image processing task of “contrast enhancement”. Contrast enhancement is an important issue in difficult and dynamic visual environments such as the ones faced by advanced driver assistant systems (ADAS). Therefore, this could help improving the image quality or the visibility in real time.

We do propose the realization or better the implementation of the coupled nonlinear oscillators’ image processing concept on top of a cellular neural network framework. Hereby, the CNN processors are viewed as a slave-system used to solve, in real-time, the nonlinear ordinary differential equations describing the coupled nonlinear oscillators’ model. The image processing based on the coupled nonlinear oscillators has a very great strong feature, which is that its processing efficiency is sensitive neither to the actual image quality nor lighting variations or states, but solely on the coefficients of the nonlinear differential equations describing the coupled oscillators’ model. The appropriate coefficients/parameters of the coupled nonlinear oscillators are determined in an offline bifurcation analysis process, which is explained further in this paper. The new resulting challenge becomes then that of being capable of solving these nonlinear differential equations in real-time. We should first notice that these differential equations (ODE’s) do have ‘constant’ coefficients, which have been selected, as explained before, from the analysis of the results of the bifurcation analysis.

Therefore, the problem setting for the CNN processor system, on top of which the coupled oscillators will be implemented, is that of solving in real-time a set of highly stiff nonlinear differential equations having constant coefficients. The input images are the frames which are then considered / taken as initial conditions for the coupled oscillator system. The real-time constraint is determined by the actual frame rate. The new key challenge becomes therefore, evidently, that of determining the appropriate templates for solving the set of stiff nonlinear differential equations. But this has been a still open issue when one looks at the actual state of the relevant literature. We could however address and efficiently solve this challenging issue in a subsequent work. The results obtained are presented in all details in another paper that we do also publish in these conference proceedings of CNNA 2010; it has the title: “CNN-based Real-time Computational Engineering” (see [28]).

The previous explanations do clearly highlight how we could combine the two concepts together in a powerful and highly efficient real-time processing framework: a *‘coupled nonlinear oscillators’ based image processing scheme on top of a CNN processors framework*”.

Contrast enhancement has been an issue of prime importance in dynamic environments. It is one of the major low-level image processing tasks needed to be done before further processing of an image can be possible at higher levels. Things become more challenging in a continuously changing environment like the one experienced by driver assistance systems on the road; weather, lighting, etc. do result in significant spatial-temporal variations of the input image quality. The real-time processing constraint does make the overall scenario more challenging: the higher the car speed, the faster the image processing must be. A continuously changing environment requires the system to be adaptive, i.e. the system should process/enhance the input image in such a way that the corresponding output image always presents/possesses the best possible contrast regardless of the effects of different environmental conditions experienced by the input image (like darkness, non-uniform lighting, raining, fog, etc.). This implies that the output image should contain significant contrast in it, so that all of the objects contained in it could be easily distinguishable by the system for further processing such as scene analysis, etc.

The implementation on top of cellular neural network of the coupled nonlinear oscillatory systems’ paradigm is a best candidate/concept for providing an appropriate answer to this need. To develop such a paradigm, a systematic analytical framework should provide tools/methods for a straight forward design and parameters calculation of a related robust and ultra-fast image processing.

The rest of the paper is organized as follows. Section 2 exploits the Routh-Hurwitz theorem to address the stability analysis of the nonlinear coupled oscillatory system. Three main states of the coupled system are depicted, namely equilibrium-, quenched-, and oscillatory- states. Analytical formulas/relations are derived under which each of these states could be displayed by the coupled system. The quality of the image ‘contrast enhancement’ is discussed in each of the possible states of the coupled system. Windows of the system-parameters are determined, under which either a good or a worst contrast enhancement can be predicted. Section 3 deals with the numerical study. An in-depth explanation of the image processing concept involving coupled nonlinear oscillators is provided. For rapid prototyping purposes a computing platform is developed, which is based MATLAB/SIMULINK. It is then used for a set of processing tasks on both images having a poor contrast and on images with very good contrast as well. Section 4 deals with the benchmarking. This benchmarking shows how far this novel approach does outperform the classical CNN based way of doing the same task, since the CNN templates used for contrast enhancement (or published in relevant books or papers) are in reality only optimal for the images used in the related training process. The later is traditionally based on offline optimization processes involving either genetic algorithms or simulated annealing or particle swarm optimization [29]-[34]. As proof of concepts of the approach developed in this paper our results are compared with those provided by the relevant literature for CNN based contrast enhancement.

We further discuss a possible implementation of the coupled nonlinear oscillators on top of a CNN computing platform. The challenge hereby is that of transforming, as much as possible, the nonlinearity types present in both ‘van

derPol' and 'Duffing' oscillators into a type of nonlinearity similar to that displayed by the elementary CNN cell. We use a novel optimization concept/process to achieve this goal. Section 5 presents a set of concluding remarks. Furthermore a summary of the key results obtained is provided.

## II. ANALYTICAL STUDY

The dynamics of a system consisting of a van der Pol oscillator coupled to a Duffing oscillator is described by the following equations:

$$\frac{d^2x}{dt^2} - \varepsilon_1(1-x^2)\frac{dx}{dt} + \omega_1^2x = c_1y + c_2\frac{dy}{dt} \quad (1a)$$

$$\frac{d^2y}{dt^2} + \varepsilon_2\frac{dy}{dt} + \omega_2^2y + c_0y^3 = c_3x + c_4\frac{dx}{dt} \quad (1b)$$

where  $c_1$  and  $c_3$  are the elastic coupling parameters, and  $c_2$  and  $c_4$  are the dissipative coupling parameters.  $x(t)$  and  $y(t)$  represent the coordinates of the coupled oscillators (i.e. van der Pol and Duffing respectively). The stability of the equilibrium points is carried out by restricting our analysis to the case where the elastic couplings (respectively the dissipative couplings) are identical. From (1), we obtain the following equilibrium points ( $c_2 = c_4 = 0$ ):

$$P_1 \left( \sqrt{\frac{c_1^2(c_1^2 - \omega_1^2\omega_2^2)}{c_0\omega_1^6}}, 0, \sqrt{\frac{c_1^2 - \omega_1^2\omega_2^2}{c_0\omega_1^2}}, 0 \right) \quad (2a)$$

$$P_2 \left( \sqrt{\frac{c_1^2(c_1^2 - \omega_1^2\omega_2^2)}{c_0\omega_1^6}}, 0, -\sqrt{\frac{c_1^2 - \omega_1^2\omega_2^2}{c_0\omega_1^2}}, 0 \right) \quad (2b)$$

$$P_3 \left( -\sqrt{\frac{c_1^2(c_1^2 - \omega_1^2\omega_2^2)}{c_0\omega_1^6}}, 0, \sqrt{\frac{c_1^2 - \omega_1^2\omega_2^2}{c_0\omega_1^2}}, 0 \right) \quad (2c)$$

$$P_4 \left( -\sqrt{\frac{c_1^2(c_1^2 - \omega_1^2\omega_2^2)}{c_0\omega_1^6}}, 0, -\sqrt{\frac{c_1^2 - \omega_1^2\omega_2^2}{c_0\omega_1^2}}, 0 \right) \quad (2d)$$

These points exist under the conditions  $c_1 < \omega_1\omega_2$  and  $c_0 < 0$  or  $c_1 > \omega_1\omega_2$  and  $c_0 > 0$ . We also obtain a critical equilibrium point  $P_c(0,0,0,0)$ . The stability of the above equilibrium points can be investigated by re-writing (1) in the following form:

$$\frac{dx}{dt} = v \quad (3a)$$

$$\frac{dv}{dt} = \varepsilon_1(1-x^2)v - \omega_1^2x + c_1y \quad (3b)$$

$$\frac{dy}{dt} = z \quad (3c)$$

$$\frac{dz}{dt} = -\varepsilon_2z - \omega_2^2y - c_0y^3 + c_1x \quad (3d)$$

and linearizing around a given equilibrium state  $(x_0, v_0, y_0, z_0)$  to obtain the Jacobian matrix  $M_J$ .

$$M_J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_1^2 - 2\varepsilon_1x_0v_0 & \varepsilon_1(1-x_0^2) & c_1 & 0 \\ 0 & 0 & 0 & 1 \\ c_1 & 0 & -\omega_2^2 - 3c_0y_0^2 & -\varepsilon_2 \end{bmatrix} \quad (4)$$

The eigen-values of the 4x4 matrix, formed from the Jacobian matrix  $M_J$  are the solutions of (5)

$$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \quad (5)$$

where the coefficients  $a_\ell$  are defined as follows:

$$a_0 = 1 \quad (6a)$$

$$a_1 = \varepsilon_2 - \varepsilon_1(1-x_0^2) \quad (6b)$$

$$a_2 = \omega_1^2 + \omega_2^2 + 2\varepsilon_1x_0v_0 + 3c_0y_0^2 - \varepsilon_1\varepsilon_2(1-x_0^2) \quad (6c)$$

$$a_3 = \varepsilon_2(\omega_1^2 + 2\varepsilon_1x_0v_0) - \varepsilon_1(1-x_0^2)(\omega_2^2 + 3c_0y_0^2) \quad (6d)$$

$$a_4 = (\omega_1^2 + 2\varepsilon_1x_0v_0)(\omega_2^2 + 3c_0y_0^2) - c_1^2 \quad (6e)$$

It can be shown (by the analysis of the oscillatory states of the coupled system and by exploiting the Routh-Hurwitz theorem) that three possible states of the system can be depicted. The first state is the quenching state (i.e. the death of oscillations). The second is the state of equilibrium. And the last one is the oscillatory state.

The system exhibits its quenching state when the critical equilibrium point  $P_c(0,0,0,0)$  is stable. It can be shown, using the Routh Hurwitz theorem, that the critical equilibrium point is stable if the following relationships are satisfied (assuming that the natural frequencies of the coupled oscillators are equal):

$$\varepsilon_1 < \varepsilon_2 \quad (7a)$$

$$\omega_1\sqrt{\varepsilon_1\varepsilon_2} < c_1 < \omega_1^2 \quad (7b)$$

It can also be shown (using the Routh Hurwitz theorem) that the non zero equilibrium points  $P_i$  ( $i=1, 2, 3, 4$ ) are stable for

$$\varepsilon_1 < \varepsilon_2 \quad (8a)$$

$$c_1 > \omega_1^2 \quad (8b)$$

Under the conditions described by (8) all the neighboring orbits of the critical equilibrium points are stable. It can be shown, using the oscillatory states analysis method (e.g. the multiple time scale method), that the coupled system displays oscillatory states. These states could be observed under the following conditions:

$$\varepsilon_1 < \varepsilon_2 \quad (9a)$$

$$0 < c_1 < \omega_1\sqrt{\varepsilon_1\varepsilon_2} \quad (9b)$$

While considering (7), (8) and (9) the coefficient  $c_1$  is used as control parameter to analyze the states of the coupled system and to depict the effects of the control parameter on the processing (here, the image contrast enhancement). When the cubic nonlinearity is chosen to be negative, the equilibrium points  $P_i$  ( $i=1, 2, 3, 4$ ) are unstable.

The next section is concerned with the numerical study aiming at verifying the analytical results obtained and also at showing some preliminary results of the processing (contrast enhancement) of images with/having very poor initial contrast. The main focus will be on showing that the quality of the contrast enhancement is different in each of the various 'parameter-windows' established analytically in (7)-(9). The advantage of this remark/feature is the possibility of predicting either a good or a worst image processing; both

depend on the selected parameter values of the coupled nonlinear oscillators' model.

### III. NUMERICAL STUDY

#### A. Description of the concept

The proposed coupled oscillatory system consists of two nonlinear oscillators, i.e., a van der Pol oscillator and a Duffing oscillator, each represented by a second order nonlinear differential equation as given in (1). From a nonlinear dynamics perspective the scheme to solve this oscillatory system is straightforward.

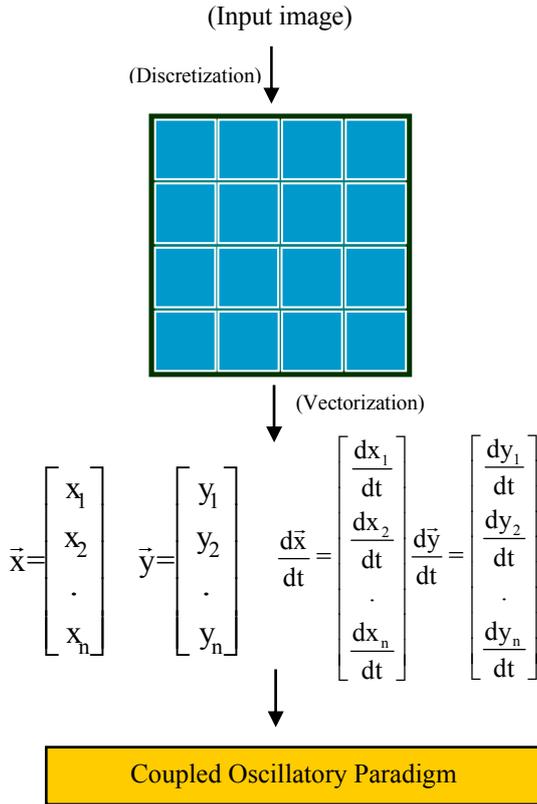


Figure 1. Image processing through the oscillatory model

In order to exploit the coupled nonlinear model/equations for some image processing tasks (e.g. contrast enhancement, edge detection, segmentation, etc.) the basic idea remains the same although a bit trickier. The input image is pixelized first, i.e., it must take a grid-like form. Then the pixelized image is vectorized. The elements of the vector image are the individual pixels. This vector image serves as initial condition vector for the coupled oscillatory system. To solve a 2<sup>nd</sup> order ordinary differential equation we need two initial conditions (i.e. position and velocity), it is the same in this case here also. To solve/process each pixel the system needs four values, which are 'position' and 'velocity' values for both the van der Pol oscillator and the Duffing oscillator. In this case, the initial conditions vector has four elements, each of which having the same size as that of the input image, i.e., two vector elements for the initial positions and two further vector elements to hold the initial velocities. The key steps of the overall principle are shown in Fig. 1.

The system generates two solutions at each time step. One is the van der Pol oscillator' solution and the other is to the Duffing oscillator one. These solutions are obtained in the form of vector images which must be converted back to the grid like shape. Normally, the input images are loaded (as initial conditions) either in  $\bar{x}$  or  $\bar{y}$  or in both. But there are different possible scenarios for initializing the model. Some of these scenarios are listed in the following:

- Loading the image in  $\bar{x}$
- Loading the image in  $\bar{y}$
- Loading the image in  $\bar{x}$  and  $\bar{y}$
- Loading the image in  $\frac{d\bar{x}}{dt}$
- Loading the image in  $\frac{d\bar{y}}{dt}$
- Loading the image in  $\frac{d\bar{x}}{dt}$  and  $\frac{d\bar{y}}{dt}$
- Loading the image in  $\bar{x}$  and  $\frac{d\bar{x}}{dt}$

The SIMULINK model (i.e. a graphical representation) that has been used for the simulations of this paper (i.e. for image contrast enhancement) is shown in Fig. 2. This graphical model is a representation of the nonlinear coupled oscillatory system from the nonlinear dynamics perspective.

#### B. Results

Our objective in this part is to connect the results obtained analytically (different states of the nonlinear oscillator system) to some sample image processing examples obtained through numerical simulations. The key issue hereby is that of establishing a correlation between the formulas derived analytically and the related image processing results obtained numerically (i.e. contrast enhancement).

It has been shown analytically that the equilibrium points (i.e. both  $P_c$  and  $P_i$ ) are stable under some analytical conditions described by (7), (8) and (9). We now want to exploit these equations to show the quality of the image processing tasks performed by the coupled oscillators' system in its equilibrium states. It is worth a mentioning that two main equilibrium states of the coupled system have been depicted analytically. The first state is the quenching state under which the critical point (i.e. the point at origin)  $P_c (0, 0, 0, 0)$  is stable. At the critical points both oscillators are mutually damped (i.e. complete damping), leading to the quenching phenomenon. When this phenomenon occurs, the result of the image processing leads to an image which is completely dark (see Fig. 3b), whatever the quality (good or worst) of the input image may be (see Fig. 3a). The following set of parameters has been used to obtain the quenching state under the conditions described in (7):  $\varepsilon_1 = 0.4$ ,  $\varepsilon_2 = 1$ ,  $\omega_1 = 1$ ;  $\omega_2 = 1$ ,  $c_1 = 0.8$ ,  $c_3 = 0.8$ ,  $c_2 = 0$ ,  $c_4 = 0$ , and  $c_0 = 0.5$

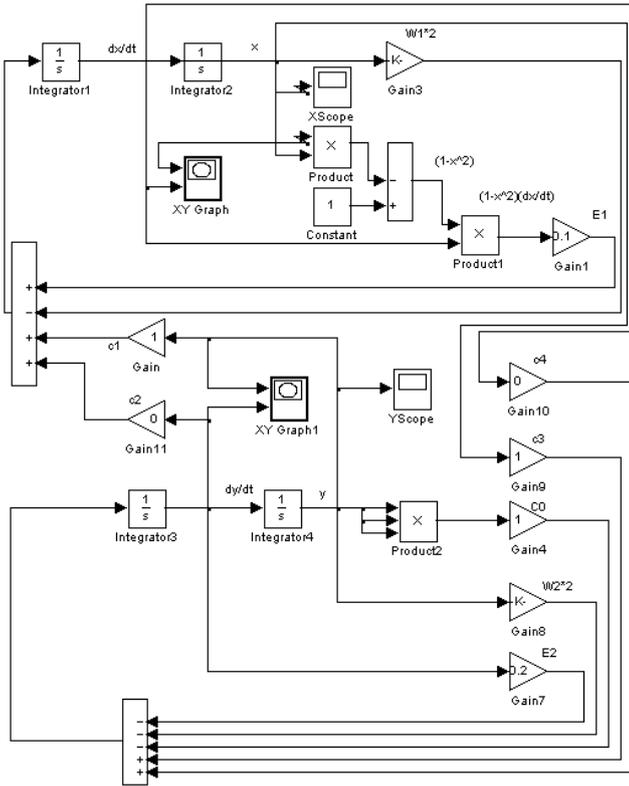


Figure 2. Simulink representation of the coupled oscillators' model



Figure 3: Result of the image processing in the case where the system is the critical equilibrium point  $P_c (0, 0, 0, 0)$ : input image (a) and result of the processing (b), leading to an output image which is completely dark (Quenching phenomenon).

An important observation which could be drawn from Fig. 4 is that the image processing quality is the highest for equilibrium points that lie much further (far away) from the critical point. For instance, the parameter values  $c_1 = 1.05$ ,  $c_1 = 1.15$ , and  $c_1 = 1.3$  lead to the following equilibrium points  $P_1(0.475, 0, 0.455, 0)$ ,  $P_1(0.923, 0, 0.803, 0)$  and  $P_1(1.52, 0, 1.17, 0)$  respectively. Therefore, by increasing  $c_1$ , the equilibrium points  $P_i$  move far away from the critical equilibrium  $P_c$  and thereby leading to a significant improvement in the quality of image processing (contrast enhancement). The results/images obtained are presented in Fig. 4. We have also performed a series of image processing numerical simulations in the 'oscillatory states' of the coupled system described by (9). Using the same set of parameters like in Fig. 4,  $c_1$  has been used as control parameter. The results obtained in Fig. 5 have revealed that in the oscillatory state of the coupled system, the quality of the processing increases with decreasing  $c_1$  (see the results of the processing in Fig. 5b, Fig. 5c, Fig. 5d).

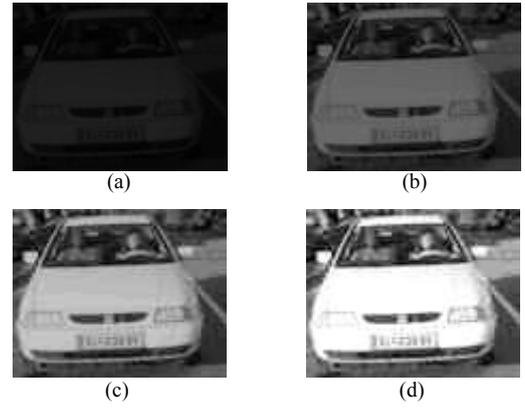


Figure 4. Effects of the control parameter  $c_1$  on the image processing quality- the system is in different equilibrium states: (a) is the input image; (b) is the related image processing result for  $c_1 = 1.05$ ; (c) is the image processing result for  $c_1 = 1.15$ ; and (d) is the image processing result for  $c_1 = 1.3$ , the later leading to the optimal result/processing obtained in the corresponding equilibrium state of the coupled system.

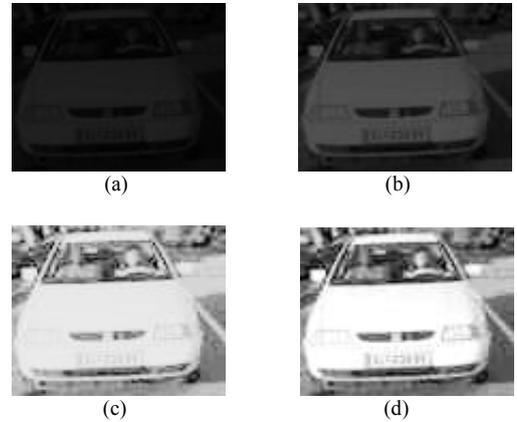


Figure 5: Effects of the control parameter  $c_1$  on the processing of quality of the input image (a) – the system is in different oscillatory states; the results of the processing are: (b) for  $c_1 = 0.6$ , (c) for  $c_1 = 0.55$ , and (d) for  $c_1 = 0.50$ , the later leading to an optimal result/processing obtained in the corresponding oscillatory state of the coupled system.

#### IV. BENCHMARKING

In this section we discuss and compare the results of the CNN based image contrast enhancement techniques published in the literature so far with those obtained through a processing by the coupled nonlinear oscillators' paradigm. A first attempt for a CNN based contrast enhancement was presented by Márton Csapodi *et al.* [22]. In this concept, another well known contrast enhancement approach, i.e. adaptive histogram equalization, has been emulated by performing a piecewise linear approximation of different mapping functions. The technique is computationally intensive since for each contextual region it requires a histogram generation, a mapping function calculations and a rescaling of pixel values according to the new mapping. Mátyás Brendel *et al.* [23] addressed the contrast enhancement problem by proposing a set of linear templates, which however do not provide good results for all test images due to the high nonlinear nature of the images. A. Gacsádi *et al.* [24] have designed a set of templates for image enhancement by minimizing the image energy function

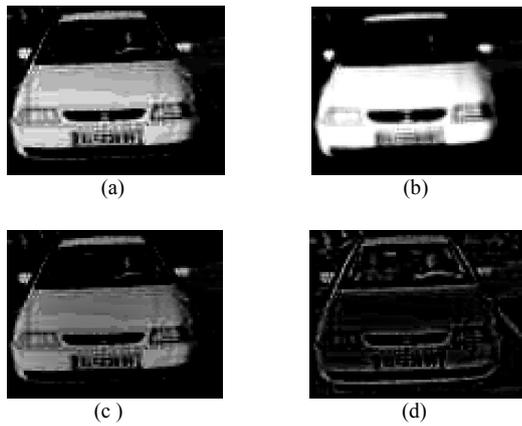


Figure. 6. Results of CNN based contrast enhancement schemes: (a) Image enhancement based on an approach developed by Márton Csapodi *et al.* [22]; (b) Image enhancement based on an approach developed by Mátyás Brendel *et al.* [23]; (c) Image enhancement based on approach by A.Gacsádi *et al.* [24]; (d) Edge preservation observed in the approach by A. Gacsádi *et al.* [24]. All these results are obtained w.r.t. the input image of Fig. 4a.

The energy function considered consists of two processes that are smoothness constraint and edge penalty. Thus, to obtain an optimum result a tradeoff between image smoothness and edge detection was to be found and adjusted. Applying the approach based on the CNN paradigm on the same input image of Fig. 4a, we have obtained an enhanced contrast w.r.t the input image but with a loss of key information (see Fig. 6). The parts of the input image with small gray level differences have been lost, e.g. driver's face, the road, the round lane and the background. In contrast to that, the optimum results (Fig. 4d, Fig. 5d) obtained through the coupled nonlinear oscillators processing paradigm clearly show that almost all of the basic information of the same input image is restored during the image enhancement processing. A comparison of Figures 5 and 6 does underscore the superiority of the coupled nonlinear oscillator based contrast enhancement while compared to CNN based approaches. The reason for the weakness of the CNN based approach lies in the essentially "supervised training/learning"-like process used to determine the templates. Due to this, the (linear) templates obtained are only optimal for the test images. Beyond that, there is no way that those templates can be optimal for images experiencing temporal and/or spatial dynamics.

Having seen the superiority of the coupled oscillators' based approach we do now propose a hybrid architecture that will combine the strong points of both CNN and the coupled nonlinear oscillators' based image processing. The core processing will be the later one. Thereby, we do propose the realization of the coupled nonlinear oscillators' image processing concept on top of a cellular neural network processors framework. Hereby, the CNN processors will play the role of a slave-system used to solve, in real-time, the nonlinear ordinary differential equations describing the coupled nonlinear oscillators' model. The image processing based on coupled nonlinear oscillators has a very great strong feature, which is that its processing efficiency is sensitive neither to the actual image quality nor to its variations or states, but solely on the coefficients of the nonlinear differential equations describing the coupled oscillators. The appropriate coefficients/parameters of the coupled nonlinear oscillators are determined, as explained in the previous sections, in an offline bifurcation analysis process. The new challenge related to the CNN processor becomes now that of being capable of solving these nonlinear differential equations

in real-time. Thus, the problem formulation for the CNN processor system is that of solving a set of highly stiff nonlinear differential equations having constant coefficients. The key challenge for this task is solely that of determining the appropriate templates. This is not trivial at all and is still an open issue if one looks at the actual state of the relevant literature. We could however solve it and we do present the related results obtained in the other paper that we publish in the proceedings of this CNNA-2010 conference; see the paper entitled "CNN based Real-time Computational Engineering" (see [28]).

## V. CONCLUDING REMARKS

CNN needs well optimized templates to perform any specific image processing task and it is well known that template optimization is still unsolved for a really straightforward and efficient CNN based computing. For dynamic environments linear templates do not provide a robust processing since every next image is different from the previous one and does in reality require a new set of appropriate templates for the same processing task. Nonlinear templates require some preprocessing to be performed on each image to get the template values that are appropriate to the actual image, leading to huge problems for real-time applications. The proposed paradigm of a coupled nonlinear oscillators based processing has shown (both analytically and numerically) domains of good/efficient contrast enhancement processing whereby the processing quality remains robust/constant and is insensitive to eventual spatial-temporal dynamics that may experience the input images. Furthermore, in CNN based processing the template-based computing involve also the pixels' neighborhood while processing each pixel. This is not the case in the nonlinear oscillators based processing paradigm as each pixel is processed independently without taking into account its neighborhood. For both paradigms, i.e. CNN and nonlinear oscillators, the input image serves as an initial condition. Both frameworks offer parallel image processing with a couple of differences: (a) CNN templates appear to be sensitive to the training conditions and lack adaptivity to dynamic environments; (b) the performance of the coupled oscillator model is independent/insensitive to both quality and dynamics of the input image.

In summary, after analyzing the results obtained we do propose the realization of the coupled nonlinear oscillators based image processing concept on top of a cellular neural network processor system. Hereby, the CNN framework will be viewed as a slave-system used to solve, in real-time, the nonlinear ordinary differential equations describing the coupled nonlinear oscillators' model. The image processing based on coupled nonlinear oscillators has demonstrated its great strong feature, which is that its processing efficiency is sensitive neither to the actual image quality nor to its variations or states, but solely on the coefficients of the nonlinear differential equations describing the coupled oscillators. The appropriate coefficients/parameters of the coupled nonlinear oscillators are determined in an offline bifurcation analysis process, which has been extensively explained further in this paper. And these coefficients remain constant and do not need to be recalculated in real-time.

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