Modeling of a Three-Echelon Supply Chain: Stability Analysis and Synchronization Issues

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***Abstract*— This paper introduces the supply chain networks integration and discusses the interest of synchronization within them. The evaluation of the causes of uncertainties within the supply chain networks is considered. The structure of a three-echelon supply chain is envisaged and the modeling of this structure is carried out. It is shown that the supply chain network can be exposed to both external and internal perturbations. The origin of these perturbations is discussed and it is shown that the stability of the supply chain network is very sensitive to these perturbations. The effects of the external perturbations on the supply chain network are considered. It is shown that these perturbations can saturate the supply chain network and also create bullwhip effects and chaotic phenomena within the network. A method is developed which is based on an adaptive algorithm for the automatic cancellation of the effects of external perturbations by re-adjusting the internal thresholds of the supply chain network in order to achieve synchronization. Some additional insights into theory and practice in supply chain managements are provided.**

Keywords—Synchronization; Supply chain networks; Supply chain network control; Supply chain optimization; Bullwhip effect; Chaos in supply chains

I. INTRODUCTION

Supply chain management (SCM) is the combination of management and science. SCM helps in improving/managing the way a company finds sources for the raw components it needs to make a product and the way it distributes the product to customers. The modern supply chain networks are global in nature and have many interconnections i.e. dependency or coupling between each. The supply chain network comprises of geographically dispersed facilities, like sources of raw materials, manufacturing plants, warehouses and transportation facilities. The facilities may be operated by single entity, many entities within a company, third-party providers or other companies. The primary objective of the supply chain network, or the individual entities in the network, is to deliver the product in the correct quantities at correct time by meeting all the quality measures with competitive price. In addition to this, the threshold of stocked products, quantities must be continuously sustained and uncertainties (that can occur within the supply chain) must be well controlled in order to avoid breaking in stocks and also the well known bullwhip effect at a given level of the supply chain. This can be achieved through a good coordination or synchronization between the various levels of supply chain.

In order to achieve the stated objective, all the stakeholders in the supply chain network are integrated/coupled. The first part of coupling is concerned with functional integration of purchasing, manufacturing, transportation, and warehousing activities. Besides functional integration intertemporal coupling, also called hierarchical integration, of these activities over strategic, tactical, and operational levels is also important [1, 2]. This intertemporal coupling requires consistency among overlapping supply chain decisions at various levels of planning. However the major role of the intertemporal integration is in product life cycle design. Improved coupling of activities across multiple companies/entities in a supply chain is a concern of increasing importance. Information technology is the key enabler for the coupling in supply chain. The information systems provide the necessary information needed for the integration. Each entity works with their own objective, there by showing a tendency only for loose coupling.

The structure of the paper is as follows, section II is concerned with a brief description of few factors that are contributing for the uncertainties and need to synchronize the uncertainties. Section III deals with supply chain modeling. We present the structure of the supply chain under consideration which is made-up of three levels. The communication between these levels and also the data traffic within them is explained /discussed and it is shown that the scenario/dynamics of the complete supply chain can be modeled by the Lorenz system. In section IV, we will describe what type of synchronization is applied in supply chain networks. Section V describes the effects of the external perturbation on the system and the controlling method. Transitions from regular effects to bullwhip effects or chaotic effects are shown through the stability analysis. Section VI deals with the conclusion and proposals for future works.

II. UNCERTAINITIES IN SUPPLY CHAIN NETWORKS

Even though integration of the stakeholders should provide the ideal solution for the optimization of the supply chain networks, in many cases it is not [1-3]. Business is not usual always; there are many things that happen with time. The major source for uncertainties in supply chain networks originates from the fact that operations are performed over long period of time and stakeholders are dispersed globally. Supply chain management is a complex set of processes and flows (information flow, product flow, monetary flow) that can either add a great deal of efficiency and profit or can cause serious problems for a company. SCM is technology intensive in some aspects, in some ways it can be compared to a simple game gossip game.

In the game, one passes the secret to his friend, which is then passed around a circle of friends. The secret can be true or false, but what is usually the case with the secret is that it becomes exaggerated – if the secret starts out as “there is an insect outside,” it ends as “a monster is in the school”.

The supply chain can often be seen as the same circle. Today’s supply chains pass information relating to item information, forecasting and replenishment upstream and downstream among manufacturers, brokers, distributors, retailers and warehouse, transportation, management and global trade systems. The final result is a flow of information from the creation of a product to the point where that product is sold to the end consumer.

As in the gossip game, the accuracy of information often depends on the quality of the data at the beginning of the cycle, and maintaining same information throughout the supply chain. If the information is wrong in the beginning, it never has a chance of being correct when it finally gets up or down the supply chain. In fact, if inconsistencies remain, there are chances for errors to propagate through the whole supply chain.

The definition of a truly efficient supply chain is when all companies involved are communicating correct or reliable data – a manufacturer is communicating the reliable product information and receiving accurate purchase orders; a retailer is receiving the specific products that were ordered; and the product is available to the end consumer at the right time and at the right price. However communicating the reliable data is always not possible in supply chains as each stake holder has different objectives and constrains. In well-managed supply chains, the inventory flows between members of the chain with little delay. The goal of supply chain management is to optimize the whole system [1-3]. Total chain-wide transportation, holding, and setup costs should be minimized. However, this integration may be difficult to achieve because different members of the chain have conflicting objectives. Supply chain members whose inventory holding cost is small, either because the capital invested is small or they can arrange financing and storage at advantageous rates, also those whose setup costs are high, will prefer large lot sizes and corresponding large inventory. Retailers with high value of goods but no setup costs will prefer to hold the minimum stock they deem necessary to ensure good customer service. These individual varying factors create demand variability in order to make maximum profits. There are also the problems of misunderstanding the market promotions, social mistrust, forecasting problems. All these leads to more inventory at different levels and can cause the well known bullwhip effect or forester effect. In this paper, the occurrence of this effect and also the chaotic effect in the supply chain due to external perturbation of the system is shown.

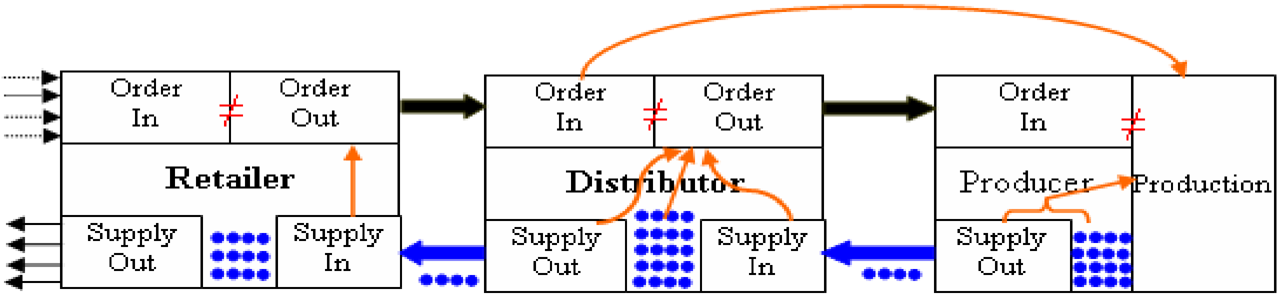

Figure 1: Three level supply chain model

III. Supply Chain Modeling

The theoretical framework for the supply chain management underlies the setting, optimization and control of the system model. Here, In this work we use the structure of a three-level supply chain as illustrated in Fig. 1.

i	Time period
m	Ratio of customer demand satisfied
k	Safety stock coefficient
x_i	The quantity demanded for products in current period
y_i	The quantity distributors can supply in current period
z_i	The quantity produced in current period depend on the order

A three echelon model is envisaged in Fig. 1 to describe a simple scenario in a very complex supply chain. Fig. 1 also illustrates the relationship between the three layers. The orders they make may not be equal to orders they receive. The order out quantity depends not only on how much inventory you have already, and how much you want to supply out. Order out quantity at retailer depends on the ratio m at which the demand is satisfied during the previous order. Distributor needs to take into consideration among other things, the distortion rate r that can occur in the inventories. The producer needs to take care about the safety stock k in order to avoid the small production batches. These phenomena are explained below with mathematical modeling.

We consider that the demand information is transmitted within the layers of the supply chain with a delay of one unit time. As illustrated in Fig. 1, the ordering quantity is not same as the requested order quantity at any level. The order quantity at current period of time at retailer is linearly coupled with the distributor and it is influenced by how much of demand is satisfied in previous period of time. This phenomenon can be represented as shown in Eq.1.

$$x_i = m(y_{i-1} - x_{i-1}) \quad (1)$$

m is the ratio at which the demand is satisfied. As it appears from Fig. 1, the dependency or coupling of distributor on producer and retailer is no more linear. Indeed the distributor needs to take the combined effect of retailer and producer into consideration before making his order, i.e., quadratic coupling. Apart from this, the distributor also needs to take into consideration the expected loss rate or distortion that can take place on the producer's supplies. This can be expressed mathematically as shown in Eq. 2.

$$y_i = x_{i-1}(r - z_{i-1}) \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

r is the distortion coefficient. The production quantity from the producer unit typically depends on the distributor's orders and the safety stock. However the distributors' orders again depend on the retailer's orders, i.e., the producer needs to take the combined effect of retailer and distributor into account before making production decisions. This can be represented as

$$z_i = x_{i-1}y_{i-1} + kz_{i-1} \quad (3)$$

Eqs. 1-3 represent the quantity demanded by customers (Eq. 1), the inventory level of distributors (Eq. 2) and the quantity produced by producers (Eq. 3).

- Where $x_i < 0$ denotes that the supply is less than customers demand in the previous period
- y_i <0 denotes that information is severely distorted and no adjustment is necessary to inventory level
- z_i <0 denotes the cases of overstock or return and hence no new productions.

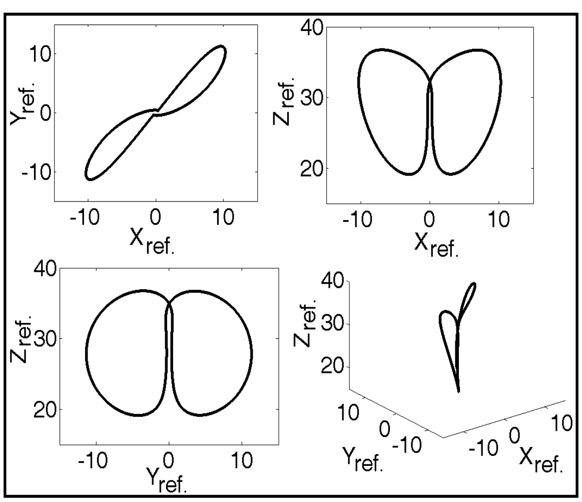

Figure 2: Reference System with given coefficients

Evidently the continuous forms of Eqs 1-3 can be rewritten as *Lorenz* equations of state:

$$\dot{x} = \sigma(y - x) \quad (4)$$

$$\dot{y} = rx - y - xz \quad (5)$$

$$\dot{z} = xy - bz \quad (6)$$

Depending upon the parameter values, this model produces a wide variety of nonlinear features. In Fig. 2, we showed the reference model with the parameter values $\sigma = 15$ $r = 29$, and $b = 2/3$ to illustrate the regular state of the reference supply chain model.

IV. Synchronization Issues in Supply Chain Networks

In this section we briefly introduce the importance of synchronization and present the modern methods to deal with it. When organizations synchronize supply chains and have real-time access to data, they enjoy several competitive advantages:

- Anomalies and exceptions are identified early and the data for intelligent response is immediately available. This greatly minimizes the bullwhip effect and saves downstream partners and customers from needless activity
- Back-end processing of change orders, modified invoices, and updates to inventory systems can happen as decisions are being made
- Visibility into partner systems makes planning easier and enables managers to see opportunities that were not apparent before. This visibility also enables supply chain partners to collaborate more effectively. This cooperation is essential when problem resolution requires the coordination of several supply chain partners
- Easier integration and expansion. Because the synchronization and data passing mechanisms are not based on packages from supply chain software vendors or ERP firms, the software is optimized for all players. Packages can be changed without disrupting the infrastructure and new partners can be added easily

Is supply chain synchronization the ultimate concept? In reality, it is only one of the steps towards the integration of all components of the supply chain. To date, the majority of supply chain efficiencies has come from improvements within the four walls of each individual company. Certainly, there has been collaboration between trading partners in terms of improved communication (EDI and current internet based web information exchanges), better information (point of sale data and the CPFR initiatives), and a general willingness to work more closely together. But the efficiencies have been gained through improvements that any executive can effect at his or her own workplace by putting in place the appropriate company-wide initiatives aimed at improving the internal business process. Now comes the most difficult part: True coordination between and among all trading partners, which is most difficult. To take the next step, it is critical that companies not only agree to communicate and work together, they must also begin to function as a single entity.

- Apart from all these data discrepancies, there may be other disturbances which occur due to external perturbations. There are different methodologies applied to study these discrepancies and to quantify their effects. In this paper we study the two major synchronization types applied in supply chain networks based on the supply chain model [4]. Complete Synchronization: the cycle time at an upstream stage equal to cycle time of the next downstream stage
- Partial synchronization: the cycle time at an upstream stage is an integer multiple of the cycle time of the next downstream stage

In this paper, we present methodology to control the effects caused by external perturbations through internal measures.

A. *Complete synchronization*

Complete synchronization enables the supply chain to react quickly to changes in demand and in product design. This type of synchronization is particularly suitable in just-in-time supply chain networks. To achieve the complete synchronization the complete chain should be integrated. Many retailers and manufacturers have looked to the Internet to provide a cost-effective avenue for improving the data deficiencies present in today's supply chains. One of the main ways to decrease the amount of inconsistent item information being passed among companies is to automate many of the item maintenance functions, such as data changes and new item introductions, performed by manufacturers. Synchronization tools automate processes that are traditionally conducted through excessive paperwork, such as informing trading partners of changes to product information. This synchronization method affects the registry and lifecycle management of synchronized products, locations and trading capabilities. Data synchronization involves a strategic combination of technologies, business strategies and implementation of industry standard in messaging that ties together the information management systems of all parties participating in a given supply chain [4, 5].

The modern synchronization tools provide automation of the paper work but not concentrate on what happen if the given data is slightly changed accidentally. Effective data synchronization first involves making information visible within a secured environment, which is again a case that we are discussing throughout this paper.

B. *Partial synchronization*

Partial synchronization is achieved through the controller item. Apart from the data synchronization as explained in complete synchronization, a controller item is developed to mitigate the effect caused due to time lag. In these synchronization major efforts is placed in modeling the effect and quantifying it. The modeling and quantification of the effect caused by the time lag, information discrepancy, and individual objectives helps in designing the controller/synchronizer element. This controller item can be unique for each entity or supply chain as a whole. Partial synchronization is also achieved indirectly by quantifying the region where uncertainties exists. After identification of uncertain regions they try to avoid or extra careful in that region [1, 3-7].

In practice, as we discussed in section II and section III, management of commercial supply chains is a challenging issue as diverse pre-defined 'data' thresholds or requirements within them should be currently stabilized with regard to the existence of some uncontrollable effects due to both internal and external perturbations to which the supply chains may be subject. The dynamic character of the information flows within the commercial supply chains is another factor which makes it very difficult to stabilize the overall behavior of the whole supply chain. Therefore, the instability of the supply chain may be directly perceived as the modification of data thresholds or pre-defined requirements based on the uncertainties discussed in section II. In section V, we will describe the effects that can be caused by the external perturbation to the stability of the system and how it can be controlled internally.

V. SYSTEM ANALASYS AND SYNCHRONIZATION

. The causes of supply chain instability can be broadly classified into two categories. The first cause is the dynamical and nonlinear character of the motions (i.e. material/products flow, information exchanges, etc…) between different entities in supply chains. The second cause originates from the effects of both external and internal perturbations to which the supply chain is subjected. An optimal management of the information flows within the supply chains may be of high importance in order to alleviate the effects resulting in negative consequences on the flows within the supply chains. This could be achieved through an adaptive control mechanism which is based on a current comparison of the dynamical data within the supply chains with the pre-defined data fixed by the requirements of the supply chains. Here, an automatic or adaptive control of the flows within the supply chains should be able to detect changes in the flows within the supply chains and act accordingly/consequently (by undertaking a given action within the supply chains) in order to alleviate the undesirable effects and therefore stabilize the system behavior that has been perturbed. The achievement of synchronization is observed when the action undertaken has allowed the recovery of the original behavior (eventually thresholds or reference requirements) of the supply chain.

This paper develops an adaptive method (algorithms and/or tools) for the systematic and automatic control of the flows within the supply chains. The external perturbations can be a consequence of the dynamical behavior of the market demand, forecasting methods, high lead times, etc. In fact, due to the dynamic changes as discussed (in time domain), some pre-defined data or requirements within the supply chains (thresholds like safety stocks) may be varying accordingly as consequence of these perturbations. The internal perturbations within the supply chains are random (or undesirable) effects which can originate from the malfunctioning of a given echelon of the supply chain (e.g. Machine breakdowns, transportation delays etc.,). It should be worth mentioning that a combination of the simultaneous effects of both internal and external perturbations may be responsible of the dynamic motion variations (e.g. flow of materials, information exchange, etc.) within the supply chain. This is a concrete and/or realistic scenario as the supply chains of many companies are currently exposed to the both types of perturbations. This later scenario is however out of scope of this paper, which does concentrate solely on external perturbations and related internal adjustment of parameters to achieve synchronization. Thus, for the sake of simplicity, we consider the case where there is no internal perturbation of the motions within the supply chains. In essence, this work looks at the possibility of re-establishing appropriate data thresholds fixed by the pre-defined behavior requirements. We restrict our analysis to the case where the external perturbations are periodic. It is well-known that periodic waves/signals are characterized by their amplitude and frequency. Considering this restriction, the amplitudes of perturbations can give/show the rate of variation of the products' quantities. The frequency of perturbations determines the rhythm or the sequence of the process related to the variations in the products' quantities.

The development of methods (algorithms) to automatically control and alleviate the effects of external perturbations is of high importance as in practice the overall system behavior may be very sensitive to external perturbations. This may lead to instability.

We develop a method based on an adaptive algorithm for the automatic cancellation of the effects of the external perturbations by re-adjusting the internal thresholds. Indeed, this method is an adaptive numerical code which performs a regulation by currently comparing the perturbed data within a given supply chain with the reference data (which are fixed and pre-defined data) according to the requirements of the three echelon supply chain. Depending upon the key characteristic parameters of the external perturbations, the state of the flows within the reference scheme of the supply chain can even be saturated due to external perturbations as shown in Fig. 3.

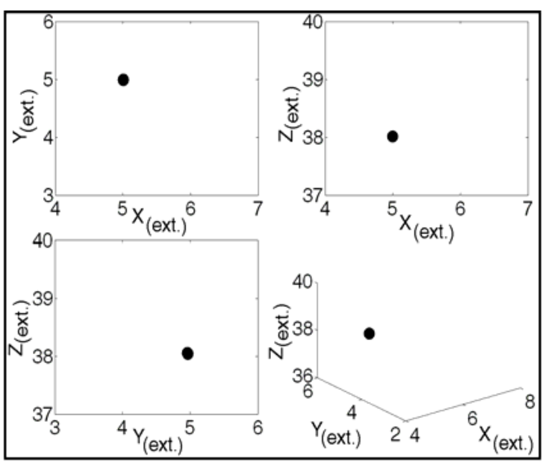

Figure 3: Saturation of the supply chain due to external perturbations

Here, the saturation manifests itself by a sudden exhibition of fixed or constant values/data along each echelon of the globally perturbed three echelon supply chain. Indeed, the states of the reference supply chain are represented by the attractors (Xref., Yref., Zref.) as shown in Fig. 2. The state of the externally perturbed system is represented by the attractors (Xext., Yext., Zext.) in Fig. 3. Fig. 3 clearly shows the saturation of the reference system due to external perturbations. To alleviate the effects due to external perturbations, we develop a numerical code based on a regulation feedback. In fact, the numerical code considers the effects of external perturbation and adjust the values of the internal parameters of the three echelon supply chain by incrementing all of them simultaneously whereby the incrementing of each internal parameter is performed in a well defined windows or interval of variation. The numerical code always compares the results from the supply chain submitted to both external perturbation and the internal adjustment of the system parameters with the results of the reference (or original) supply chain behavior. A threshold error was fixed (which is less than approximately 0.1) under which full alleviation of the effects due to external perturbation is supposed to be effective, leading to the achievement of synchronization, which results in the recovery of the reference system behavior. This is clearly illustrated in Fig. 4 where the attractors obtained are almost similar to those of the reference model shown in Fig.2.

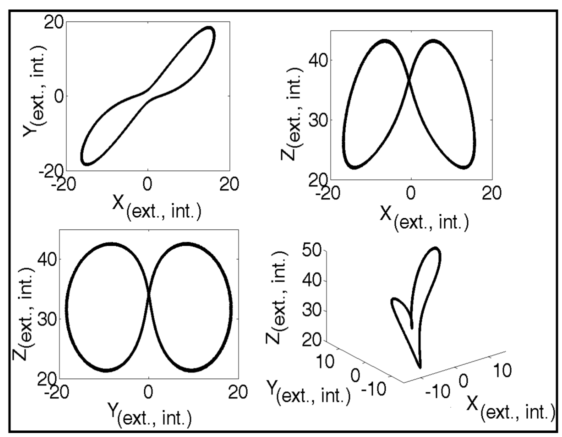

Figure 4: Alleviation of the saturation effect caused by external perturbation with adaptive numerical code

In a further study, we considered the same reference model of the three echelon supply and show that a particular change of the key parameters of the external perturbations can lead to well- known bullwhip and chaotic phenomena. This is clearly illustrated in Fig. 5 which shows chaotic attractors representing the perturbed state of the supply chain.

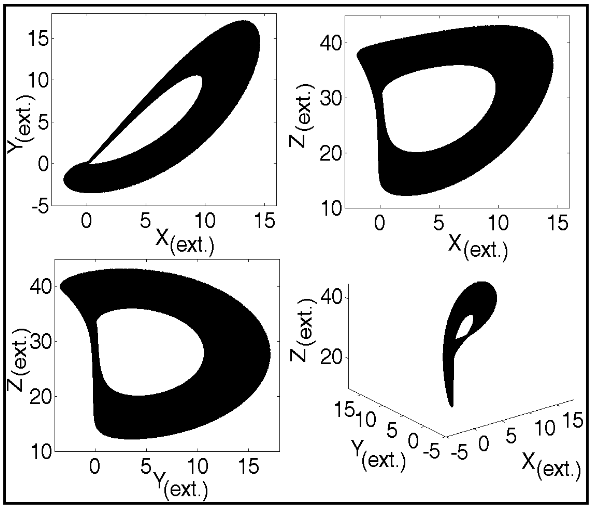

Figure 5: chaotic state of the supply chain due to external perturbations

For these specific states of the perturbed supply chain we obtained that the value of the Largest 1D numerical Lyapunov exponent is equal to +0.032. The same feedback/regulation process (see the description above) has been performed and we obtained the achievement of the complete synchronization as shown in Fig.6.

The adjustment of some internal parameters could re-establish the reference supply chain system behavior as shown in Fig. 6. This last case shows the achievement of synchronization with very small values of synchronization errors as illustrated by the attractors in Fig. 6 which are similar to those in Fig. 2.

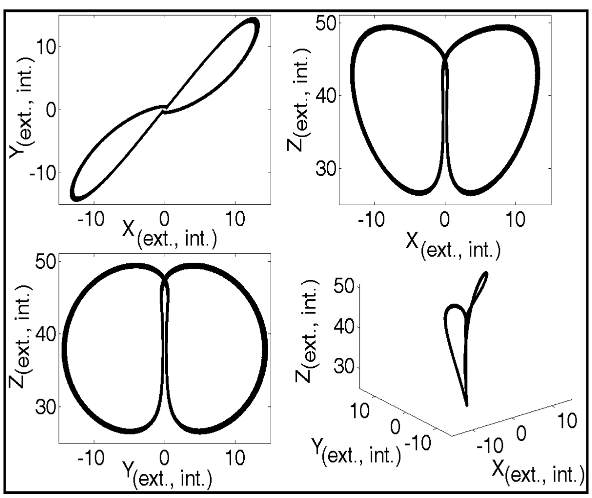

Figure 6: Alleviation of the chaotic effect caused by external perturbation with adaptive numerical code

VI. CONCLUSION

The management of supply chain networks is a complex issue which involves numerous dynamic situations varying with time. In addition to the coupling between the entities, the entire supply chain is customer oriented. In this modern era of globalized supply chains, market demand is short lived, forecasting methods are prone to dynamics, service contracts are well fixed in advance and many other constrains create uncertainties as discussed throughout this paper. This paper sets up a three-level supply chain, similar to the well known *Lorenz model*, and probes into the causes and presence of the saturation and chaos when the system receives external perturbations. This work illustrates how a supply chain can go into saturation mode under external perturbations. It also details how the same system can go to chaotic mode causing forester effect, if the parameters of the external perturbations are changed. The states illustrated in this work can occur to different parameter values i.e., they are not specific to only a set of values. This paper also proposes an adaptive algorithm for the automatic cancellation of the effects of the external perturbations by re-adjusting the internal thresholds. This method is particularly appealing as it is possible to control or adjust the internal thresholds of the supply chain network. The achievement of synchronization was shown for different sets of the threshold parameters. The simulation results show that the proposed methods are indeed effective. The solutions proposed in this paper offer a new range of possibilities for risk managers and provide a future research direction. The future research is also proposed in arriving at the adaptive synchronization with the help of analog simulation based on CNN (Cellular Neural Network) technology. An interesting and challenging problem of practical interest may be the development of adaptive numerical codes to achieve synchronization within a supply chain network under the cumulative effects of both internal and external perturbations. This is a realistic scenario which currently manifests itself in commercial supply chain networks. It would also be of great interest the analysis the coupling between different structures of supply chains and the achievement of synchronization within them. It is well known that in practice, many commercial supply chains are coupled.

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