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Contribution to Modeling and Simulation of Supply Chain Networks from a Nonlinear Dynamics Perspective

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Klagenfurt, am 02.11.2009

MSc Koteswara Rao Anne
DEDICATION

This dissertation is dedicated to my parents who offered me unconditional support, encouragement and love. A special feeling of gratitude to my late father, A.V.K. Prasad, who emphasized the importance of education and gave that special gift to me.

I would also like to dedicate this dissertation to two of my grandmothers, Mrs. Padmavathi Anne and Mrs. Sarojini Devi Tottempudi who have raised me to be the person I am today.
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ABSTRACT

For a long time the supply chain was considered as a linear system where raw material entered at one end and finished goods exited from the other. Each entity of this supply chain used to work in isolation and hold large inventories and excess capacity in order to make them more resilient to both internal and external variability and volatility. Today’s global marketplace is increasingly dynamic and volatile. This dynamic and volatile nature produces various types of uncertainties along the supply chain network, the most important being demand uncertainty, supply uncertainty, delivery uncertainty, and forecasting uncertainty. The management of supply chain networks involves many processes that are either interlinked or loosely coupled. And many of these processes are particularly sensitive to the effects of the mentioned uncertainties. To reliably cope with these sensitivities pose many research challenges that need to be addressed in order to provide solutions that can aid in the efficient management of the supply chain networks at different stages of the decision making process: strategic, tactical and operational. Traditionally, companies in the SCN have relied on decision based on both experience and intuition while facing uncertainties. Decisions made on the basis of intuition can however cause the supply chain to exhibit various nonlinear dynamical behaviors.

This thesis does in essence answer the following seven research questions related to the effects of external uncertainties in the supply chain network:

- What are realistic/appropriate performance metrics for supply chain networks with regards to the high dynamics of the external environment they are embedded in?
- What are the limitations of the traditional modeling and simulation methods of the supply chain networks?
- How far can methods/instruments from the area of nonlinear dynamic systems be used to efficiently model, simulate, and manage/control supply chain networks?
- What are the potentially reachable improvements due to the use of appropriately selected methods and tools from nonlinear dynamics in supply chain network?
- How far can appropriate nonlinear control methods be adapted for the control of supply chain networks in order to stabilize them despite external perturbations?
- How far can the theoretically proven efficient use of methods and tools from nonlinear dynamics be validated on a real SCN case study?
- How far can appropriate concepts from nonlinear dynamics be useful in reducing the complexity of the real time planning processes within supply chain network with particularly strict performance requirements (i.e., the so-called Lean supply chain networks)?

Appropriate modeling and simulation is the key for a better and sharp understanding of the complex behavior of the "nonlinear" dynamical systems called "supply chain networks". This further enables the design of appropriate strategies and methods to improve their performance. Of particular importance is the understanding of their susceptibility to both indigenous and exogenous fluctuations. The traditional SCN modeling approaches (such as "agent technology" and "petri-nets", etc.) have been shown to have limited ability to accurately quantify the effects of uncertainties along the supply chain, both in upstream and downstream. Appropriate modeling and simulation further provide a solid basis for the design of appropriate control concepts for the mitigation of the effects of those uncertainties. In this thesis, the "coupled oscillatory
systems" based modeling method from the dynamic systems theory has been proposed as an appropriate modeling paradigm of supply chain networks. It does enable a better investigation of complex resulting behaviors of external uncertainties. Related mathematical models have been derived whereby all important parameters that can affect the key performance indicators of the SCN have been considered and exploited. In the core, the "coupled oscillator model" we have designed is mainly similar to the very popular "Lorenz system" which is known to be highly sensitive to initial conditions as well to both external and internal perturbations.

After the appropriate modeling of supply chain network through a coupled oscillator model, two important analysis methods from the nonlinear dynamic systems theory, namely "phase portrait analysis" and "bifurcation analysis" have been used to extensively analyze all possible behaviors of an abstract and theoretical reference SCN network subjected to external uncertainties. The form and structure of the phase portrait of the SCN subjected to external uncertainties has been observed to be varying from a "point" up to "discernible patterns", thereby indicating that the supply chain may be experiencing different states such as regular states, periodic states, saturation, and even chaos. Therefore, the new modeling approach paired with the two analysis instruments have demonstrated their capability to really assist in better simulating and understanding the highly complex nonlinear behavior of real supply chain networks.

After this, the next challenge has been that of designing a control strategy to stabilize the supply chain network against external perturbations, especially when these could lead its behavior into a chaotic state. The "bifurcation analysis" has been proven through diverse studies we have conducted to be capable of precisely showing in which parameter regions the supply chain network may experience instability and chaos. The bifurcation analysis is helpful in the process of deriving or determining different parameter ranges in which the SCN is either stable or rather instable or even chaotic.

Concerning the stabilization of the supply chain network against external perturbations and/or uncertainties, a control strategy based on adjusting internal parameters of the supply chain network has been developed and demonstrated. The results have proven the capability of a scheme based on the so-called "active control algorithm" to adjust internal parameters in order to mitigate the effects of uncertainties by bringing back the SCN system from an instable or chaotic behavior to the reference normal behavior. The canceling or alleviation of the effects due to external fluctuations has been successfully demonstrated.

The preciousness of this novel modeling, analysis and control approach based on nonlinear dynamics concepts is very appealing especially in a time of innovative products with short product life cycles. Therefore, this dissertation considers an appropriate case study that is the virtual pet toy (Tamagotchi™) supply chain network to illustrate the usefulness for decision makers of performing a thorough bifurcation analysis of a given supply chain network. The "Tamagotchi™ supply chain network" scheme has been taken from the relevant literature. After appropriate modeling, a systematic bifurcation analysis has been carried out on the model and the results have clearly illustrated that the knowledge and insights provided by this analysis should have been useful for appropriate strategic decisions to avoid the tremendously unfortunate effects that this supply chain network has experienced especially, for example, the huge losses despite the tremendous success of the product.

The management of a special type of supply chain networks with strict/high performance requirements, called "lean supply chain networks", involves huge real time planning and re-planning processes especially due
to the inherent dynamics of the real external environment. Just-in-time manufacturing schemes are examples of some of the publicly well-known concepts related to such SCN concepts. Much of those (re-)planning processes are essentially a mater of resource (re-)allocation and scheduling. Such problem settings are generally computationally NP-hard. Thus, speeding-up the real-time planning processes in such “lean supply chain networks” is a key challenge. The nonlinear dynamics area has provided inspiration to developed a novel ultra-fast scheduling scheme involving an emulated "analogue computing" concept based on cellular neural networks. This new concept has proven to be scalable, as its computational complexity grows linearly against problem size. It therefore clearly outperforms competing traditional approaches, which have a rather, in the trends, exponentially growing complexity. The extremely high speeding-up of the scheduling processes does also enable a better reaction of the supply chain to fast internal or external changes and thereby significantly contribute to ensuring the stabilization of its overall performance and behavior.
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Chapter 1

Introduction

1.1 Motivation and general context

Supply chain network (SCN) management has been defined as the management of upstream and downstream relationships with suppliers and customers in order to create enhanced value in the final market-place at less cost to the supply chain as a whole [14]. The costs involved in supply chain may include many number of items depending upon the business, some of these costs are summarized as per the AMR Research opinion as following:

- Material acquisition costs.
- Order management costs.
- Customer service costs.
- Inventory management costs.
- Planning and forecasting costs.
- Inter-facially transportation costs.
- Inbound and outbound transportation costs.
- Facility investment and maintenance costs.

Even though the major objective of SCN management is to reduce the costs, the other key objectives include customer satisfaction and to be competitive in the market-place. The SCN management plays a very important role in the product’s success in the market-place as today’s products heavily rely on components/assembly/production and delivery from suppliers around the globe. In today’s global market-place with shrinking profit margins, fierce competition,
short life cycle nature of products and high level expectation of customer satisfaction/support before and after buying the product are emphasizing the need for better management of supply chain networks. The potentials of efficient supply chain network management are enormous and diversified into many areas:

- Grocery industry in USA is estimated to save $30 billion (10% of operating cost) by using effective logistics and supply chain strategies [15].

- Volkswagen of America (VoA), a wholly owned subsidiary of Volkswagen AG, with 750 dealer sites across USA estimated in 1997, it can save approx. $20 million annual savings in transportation related costs by implementing effective supply chain management policies [16].

- Mahindra and Mahindra, which produces farm equipment in India with 400 plus dealers and 800 plus suppliers, could reduce the replenishment lead times by more than 65% with efficient supply chain management [17].

- National Semiconductor after modifying their supply chain practices in 2004 could increase 34% in sales and decrease 47% in delivery lead time [18].

- Compaq estimated that it had lost $0.5 billion to $1 billion in sales in 1995 because laptops were not available when and where needed [18].

- Toyota was forced to shut down half of its assembly lines for six weeks following a fire at its key supplier. This disruption costed Toyota an estimated $40 million per day [19].

- The ten-day shutdown of 29 ports in the US in 2005 is reputed to have cost the US economy $1 billion a day [20].

Researchers have proposed modeling and simulation approaches with focus on optimizing some of the processes involved in managing the SCN. SCN management involves many processes that are interlinked or loosely coupled and many processes are prone to the effects from the external dynamics of the market-place. These complexities pose many research challenges that need to be addressed in order to provide solutions that can aid efficient management of the SCN at different stages such as strategic stage, tactical stage and operational stage.

1.2 Problem statement

The objective of any player in a supply chain network is to achieve maximum profits and to give maximum customer satisfaction. In addition to the stated individual objective, all
1.2. PROBLEM STATEMENT

players involved in SCN also have a responsibility towards the global objective of resilient and competitive SCN.

Christopher [14] emphasizes this by stating:

"Competition in the future will not be between individual organizations but between competing supply chains"

This realization has made the entities in the SCN to look beyond their own boundaries to assess how the resources of each other can be utilized to achieve the global objective without compromising on their own objective. Today’s global market-place is increasingly dynamic and volatile. This dynamic and volatile nature produces various types of uncertainties along the SCN, for example, demand uncertainty, supply uncertainty, delivery uncertainty, and forecasting uncertainty. The major source for uncertainties in supply chain networks originates from the fact that operations are performed over long period of time and stakeholders are dispersed globally. Apart from these uncertainties caused by external sources, uncertainties are also observed on a daily basis (for example machine breakdowns, wrong supplies and supply shortages.) making SCNs’ as complex systems. The uncertainties propagate along the upstream and downstream of the entire SCN leading to the production of various non-linear dynamic effects. In addition to these uncertainties, the relationship between various players in the supply chain is often characterized by mistrust and competition.

Moreover, inventory is generally used as insurance against the uncertainties. In the case of a single enterprize-based SCN it is relatively easy to overcome the uncertainties with properly sized inventories at each stage such as raw materials, work in process and finished goods inventories. The present day statistical tools and forecasting methods can satisfactorily aid in determining how much must be held to satisfy the customer demand for the particular product despite the uncertainties [21]. However, the problem is much more complicated when considering the whole network consisting of different players distributed globally. Practically each player holds some inventory to protect against uncertainties, but the real difficulty is in determining how much must be held and where to hold it. Researchers have identified better inventory management as one of the key to deal better with uncertainties and proposed different inventory modeling approaches based on mathematical programming models.

However, the efficiency of the SCN lies much beyond the scope of inventory management. This realization led to the development of models to visualize the operations of entire SCN using visual modeling and simulation methods such as system theory. These visual modeling
approaches shed light on the importance of the forecasting models, resource utilization and allocation relationships, transportation models and the importance of the information sharing. With the advancement of Information Technology (IT) and the emergence of Enterprise Resource Planning (ERP) systems, many of these individual models are integrated into enterprise IT solution packages to provide the intelligent integrated solutions to manage the SCN. These solutions and approaches helped companies to achieve better results as shown in section 1.1, but they are still not able to mitigate the effects caused by the external uncertainties [21, 22, 23, 24].

To date, there is no clear analytical way to calculate the propagation of uncertainties up and/or down the supply chain. Traditionally, firms have relied on the experience and intuition in facing uncertainties. The decisions made with intuition can make the supply chain exhibit various nonlinear dynamic states. The beer game developed at MIT to introduce students and industrialists to the concepts of economic dynamics shed further light on supply chain dynamics [25]. The game, a simple case of the SCN with manufacturer, distributor and retailer, shows the effect of the inter-relating feedback from the loosely coupled players on the SCN dynamics. Even though the simulation is run only for 60 weeks, which is quite less than the fundamental period of the system, it has been found that one in four management teams in the supply chain creates deterministic chaos in the ordering pattern and inventory levels [22]. These results also showed that the slightest policy change or error such as delayed order, manager forgetting something and order input error could result in a stable system going into chaotic region. This clearly demonstrated in practice the occurrence of chaos in supply chains. This is the major motivation behind this thesis to study the applicability of the concepts/tools from nonlinear dynamic theory to manage the SCN.

1.3 Research questions and objectives of the thesis

This thesis in essence addresses the following research questions to

- What are realistic/appropriate performance metrics for supply chain networks with regards to the high dynamics of the external environment they are embedded in?

- What are the limitations of the traditional modeling and simulation methods of the supply chain networks?

- How far can methods/instruments from the area of nonlinear dynamic systems be used to efficiently model, simulate, and manage/control supply chain networks?
1.3. RESEARCH QUESTIONS AND OBJECTIVES OF THE THESIS

- What are the potentially reachable improvements due to the use of appropriately selected methods and tools from nonlinear dynamics in supply chain network?
- How far can appropriate nonlinear control methods be adapted for the control of supply chain networks in order to stabilize them despite external perturbations?
- How far can the theoretically proven efficient use of methods and tools from nonlinear dynamics be validated on a real SCN case study?
- How far can appropriate concepts from nonlinear dynamics be useful in reducing the complexity of the real time planning processes within supply chain network with particularly strict performance requirements (i.e., the so-called Lean supply chain networks)?

In the following we elaborate more on the research questions.

**What are realistic/appropriate performance metrics for supply chain networks with regards to the high dynamics of the external environment they are embedded in?**

Performance metrics are treated very differently in industrial environment especially in supply chain networks. Supply chain networks are amalgamation of many individual ordinations. These individual organizations have individual performance metrics. However, as the competition is shifting from individual organizations to supply chain networks, supply chain network wide performance measurement and metrics have received much attention from researchers and practitioners. Performance measurement and metrics have an important role to play in setting objectives, evaluating performance, and determining future courses of actions. The selection of the realistic/appropriate performance metrics for supply chain networks with regards to the high dynamics of the external environment they are embedded is important to study/evaluate the modeling and simulation approaches.

**What are the limitations of the traditional modeling and simulation methods of the supply chain networks?**

Appropriate modeling and simulation is the key for a better and sharp understanding of the complex behavior of the "nonlinear" dynamical systems called "supply chain networks". This further enables the design of appropriate strategies and methods to improve their performance. There are a number of SCN modeling methods that have been proposed and developed over the years depending upon the requirements. Each modeling approach has its own advantage and disadvantages. The selection of the modeling method also depends on the requirements of the application. In this thesis, the main focus is on identifying the suitable modeling approach to detect the nonlinear dynamics presented in the SCN when subjected to uncertainties.
CHAPTER 1. INTRODUCTION

How far can methods/instruments from the area of nonlinear dynamic systems be used to efficiently model, simulate, and manage/control supply chain networks?

This important question is the basis for this thesis. The theory of dynamic systems provides methods for modeling and studying the phenomena that undergo spatial and temporal evolution. SCN do also experience similar phenomena as the relations and the dynamics between different players in the supply chain network undergo the spatial and temporal evolution. The effects of the market place uncertainties also manifest over the time and from different players into the system. The concepts from the dynamic system theory provide robust methods/tools to analyze the behavior of complex systems over time and space. A dynamical system consists of mainly three major components, namely [26]:

- The state space or a setting in which the dynamical behavior takes place.
- Mathematical rules which specifies the evolution of the system in time.
- Initial condition or the initial state from which the system starts.

Interestingly SCNs are also dynamical systems. Therefore, the use of methods from dynamic systems in managing the SCNs is evident; however an appropriate adaptation taking the specific context into account is necessary.

What are the potentially reachable improvements due to the use of appropriately selected methods and tools from nonlinear dynamics in supply chain network? Dynamic systems theory provides many methods for modeling and simulation of the dynamic system. The oscillatory systems based modeling approaches are particularly relevant to the modeling of SCNs as they are sensitive to initial conditions and external and internal perturbations. One of the important concepts from the dynamical system theory that has high potential for analyzing the SCN is the theory of stability. Stability theory concerns the behavior of the systems under various dynamic changes such as change in initial conditions, system parameter variation, system uncertainties and exogenous uncertainties. The dynamical system theory provides different tools such as ”bifurcation analysis” and ”phase space analysis” to describe the state of dynamic system. The stability theory concept is particularly needed in order to understand the effects of market-place fluctuations: demand fluctuations, supply fluctuations and delivery uncertainty on the SCN. In this thesis, the usability of both ”bifurcation analysis” and ”phase space analysis” in analyzing the SCN evolution under uncertainties will be considered.
1.3. RESEARCH QUESTIONS AND OBJECTIVES OF THE THESIS

How far can appropriate nonlinear control methods be adapted for the control of supply chain networks in order to stabilize them despite external perturbations?

The stability theory helps to understand the effects of uncertainties on the SCN. It is important to mitigate the negative effects caused by the uncertainties. Traditional approaches to mitigate the effects caused by uncertainties are not effective as explained in section 1.2. This motivated to look for alternate approaches in dynamic systems theory that can aid in stabilizing the SCNs despite of perturbations. The modern approaches in the SCN domain relay on IT tools to cancel or avoid the negative effects of the uncertainties. The IT based tools focus more on data synchronization but not on controlling the effects of uncertainties. We have seen in section 1.2 the MIT developed beer game simulation exhibits chaotic behavior under the influence of external uncertainties despite the usage of IT tools. In order to control the chaos and other non deserving behaviors, in this thesis nonlinear control and synchronization approaches are examined to control and synchronize the SCN.

How far can the theoretically proven efficient use of methods and tools from nonlinear dynamics be validated on a real SCN case study?

After analyzing the potentials of exploiting the methods/tools from dynamic systems theory in a theoretical supply chain model consisting of manufacturer, distributor and retailer, it is envisaged to apply the methods on a case study example. The application of the methods/tools on a real case study example is also one of the key objectives of this thesis. The case study is carefully selected subjected to the following criteria:

- The supply chain network should be dynamic and complex.
- The supply chain network is prone to external fluctuations and has clear evidence that it is effected due to the lack of necessary tools.
- Combines the effects of several phenomena such as information distortion, bullwhip effect, boom and bust, and multi-echelon decisions.
- The relations and dynamics between the different players in the SCN are clearly documented and previously validated.
- A representative case.

The above criteria guarantees that we are performing analysis on the fit case. In this thesis, short life cycle products supply chain networks are especially considered as they are more
dynamic, complex and provide very less time space to react to the external fluctuations. These SCNs’ demand better planning before the launch of the product by considering the eventual external fluctuations. Stability theory based analysis is particularly interesting in managing these SCNs’ as we can visualize various dynamic states it exhibits when subjected to external fluctuations for different parametric values.

How far can appropriate concepts from nonlinear dynamics be useful in reducing the complexity of the real time planning processes within supply chain network with particularly strict performance requirements (i.e., the so-called Lean supply chain networks)?

The management of a special type of supply chain networks with strict/high performance requirements, called "lean supply chain networks", involves huge real time planning and re-planning processes especially due to the inherent dynamics of the real external environment. Much of those (re-)planning processes are essentially a mater of resource (re-)allocation and scheduling. Such problem settings are generally computationally NP-hard. Thus, speeding-up the real-time planning processes in such "lean supply chain networks" is a key challenge. Lean SCNs which require elimination of "waste" at many of its processes such as transportation, production, and order fulfillment requires real time planning and re-planning. The major technologies that contributed to solve these problems vary from algorithms from operations research to most sophisticated searching algorithms and artificial intelligence(AI). However, these approaches have limitations in terms of NP-hard and local minima. The methods from the dynamic systems provide the concepts that are computationally efficient and fast. In this thesis, cellular neural networks based "analog computing" tools are especially considered as they are computationally efficient and fast.

1.4 Summary of the key contributions of the thesis

It is observed from the literature that the entities in the SCN are coupled, entities such as customer and retailer are loosely coupled (no fixed contracts or relationships) and entities such as manufacturer and distributor are tightly coupled (fixed contracts), and the effects of uncertainties propagate in both upstream and down stream. This makes the primary objective of the modeling approach to include the market induced uncertainties into the model, to quantify the effects of uncertainties and to mitigate the effects. State-of-the-art modeling methods in SCN have been studies by classifying them into modeling frameworks by standard organizations and simulation based modeling approaches that are developed to analyze and optimize SCN.
Different modeling approaches are compared over a set of comparison criteria, such as ability to quantify the dynamics, ability to deal with uncertainties, scalability and adaptability of the models, to list their merits and demerits. The traditional modeling approaches such as system theory, agent technology and petri-nets have limited ability to quantify the effects of uncertainties along the supply chain, both in upstream and downstream, and limited scope for the mitigating the effects of uncertainties. The mathematical programming based modeling approaches especially depend on the clear definition of variables. To some extent discrete event simulation modeling approach considers the dynamic nature of variables. The details of the finding are listed in chapter 4.

In this thesis, the "coupled oscillatory systems" based modeling method from the dynamic systems theory has been proposed for the investigation of the effects of external uncertainties. A simple SCN consisting of manufacturer, distributor and retailer (three levels) is considered for modeling the generic SCN that can represent the broad SCN principles. Related mathematical models were derived by exploiting and considering important parameters that can effect the key performance indicators of the SCN. The dynamics of this SCN were modeled mathematically by the well-known Lorenz oscillator. From the theory of nonlinear dynamic systems it is proved that this model produces a wide variety of nonlinear features depending upon the parameters’ values. This model is particularly of interest when dealing with the modeling of scenarios/phenomena which are very sensitive to initial conditions and to both internal and external uncertainties as well. An example SCN is defined by assigning the optimum values for the key parameters in the SCN model to consider it as a reference SCN model for the further analysis. The description of the model and the mathematical formulation of the model are described in chapter 6.

After the appropriate modeling of supply chain network through a coupled oscillator model, two important analysis methods from the nonlinear dynamic systems theory, namely ”phase portrait analysis” and ”bifurcation analysis” have been used to extensively analyze all possible behaviors of an abstract and theoretical reference SCN network subjected to external uncertainties. The form and structure of the phase portrait is used to reveal the information about the supply chain behavior for the chosen parameter values under the influence of the external fluctuations: in case of the form of a point, the system is in saturation; in case of a periodic orbit, the system is in a regular state; and in case of discernible patterns, the system is in a chaotic state. The example supply chain phase portrait reveals a periodic orbit indicating that the example system is in the regular state before subjecting to the external fluctuations. However, after subjecting to the external fluctuations, the form and structure of the phase portrait varied from a point to discernible patterns indicating that the supply
chain is going through different states such as saturations and chaos. The other important method that gives more broader view on the stability of the system over a wide parameter range is bifurcation analysis. The bifurcation analysis is carried out for two important internal parameters (i.e., rate of customer demand satisfaction and rate of information distortion at distributor) to illustrate the effectiveness of the bifurcation analysis. It has been found through bifurcation diagrams that the effects due to fluctuations can lead to both chaotic and regular states of the supply chain and that these states alternate when monitoring the internal parameters of the supply chain. The bifurcation analysis in this work has been shown to be necessarily important as it could help the decision makers at various levels such as strategic, tactical and operational levels, to better understand the performance of the supply chain over a range of parameter space. Therefore, the new modeling approach paired with the two analysis instruments have demonstrated their capability to really assist in better simulating and understanding the highly complex nonlinear behavior of real supply chain networks. The details of these two methods and the significance of these methods is explained in chapter 6.

After this, the next challenge has been that of designing a control strategy to stabilize the supply chain network against external perturbations, especially when these could lead its behavior into a chaotic state. Concerning the stabilization of the supply chain network against external perturbations and/or uncertainties, a control strategy based on adjusting internal parameters of the supply chain network has been developed and demonstrated. A control scheme is conceptualized and designed based on ”active control algorithm” to cancel or alleviate the effects due to the external fluctuations. It is also shown that this cancelation or alleviation leads to the achievement of synchronization which is characterized by the re-establishment of the reference example supply chain. The control process with active control algorithm is based on the variation of the internal parameters of the supply chain in well-specified ranges obtained thorough the bifurcation analysis. These are ranges within which the achievement of regular state is possible and the range is acceptable for all the stakeholders of the supply chain. The details of the active control algorithm and its suitability to SCNs is described in chapter 7.

The preciousness of this novel modeling, analysis and control approach based on nonlinear dynamics concepts is very appealing especially in a time of innovative products with short product life cycles. The short life cycle character of these products demands understanding of the affects of external fluctuations on the supply chain even before they start the production as the efficiency and performance of the supply chain is directly related to the success of the product. By considering this, in this dissertation as a case study virtual pet toy, Tamagotchi™, products’ supply chain is considered to illustrate the usefulness of performing bifurcation
1.4. SUMMARY OF THE KEY CONTRIBUTIONS OF THE THESIS

analysis on the supply chain. Tamagotchi™ was the first of the virtual pet games, introduced in 1996 by the Japanese toy manufacturer, Bandai Co. This case provides a good example illustrating the problems that can arise from the interactions between unpredictable demand, boom or bust, and capacity decisions in the short product life cycle setting. Even though the product Tamagotchi™ is a tremendous success in the market, the Bandai Co lost 16 billion Yen in 1998. Through bifurcation analysis we have showed how Bandai Co could have avoided the losses by analyzing the system over a set of parameter space. The details of the case study and the bifurcation analysis to the Tamagotchi™ SCN are explained in chapter 8.

The management of a special type of supply chain networks with strict/high performance requirements, called "lean supply chain networks", involves huge real time planning and re-planning processes especially due to the inherent dynamics of the real external environment. Just-in-time manufacturing schemes are examples of some of the publicly well-known concepts related to such SCN concepts. Much of those (re-)planning processes are essentially a mater of resource (re-)allocation and scheduling. Such problem settings are generally computationally NP-hard. Thus, speeding-up the real-time planning processes in such "lean supply chain networks" is a key challenge. The nonlinear dynamics area has provided inspiration to developed a novel ultra-fast scheduling scheme involving an emulated "analogue computing" concept based on cellular neural networks. This new concept has proven to be scalable, as its computational complexity grows linearly against problem size. It therefore clearly outperforms competing traditional approaches, which have a rather, in the trends, exponentially growing complexity. The extremely high speeding-up of the scheduling processes does also enable a better reaction of the supply chain to fast internal or external changes and thereby significantly contribute to ensuring the stabilization of its overall performance and behavior.

1.4.1 Scientific significance of the thesis

The theory and methodologies of the dynamic systems are applied to understand and address the effects of uncertainties and volatilities in SCNs’. The coupled oscillatory systems based modeling approach gives a new direction in modeling the dynamics of the SCN. Stability theory based analysis of the SCN using bifurcation analysis and phase space portraits provides a new research direction as they help to understand the dynamics of the critical parameters of a supply chain more precisely. This will also open doors to investigate the critical problems in SCNs’ through the other methods/tools from nonlinear dynamic systems theory. The proposed control and synchronization methods using nonlinear control theory gives insights into mitigating uncertainties effects that may occur even at many isolated systems such as
transportation planning systems, inventory management systems and forecasting systems. The resource allocation approach using cellular neural network opens the doors for new research avenues in mathematical programming for optimization.

1.4.2 Practical significance of the thesis

The stability theory based analysis of the SCN using bifurcation analysis and phase space portraits proposed in this work has been shown to be of necessary importance as it could help the decision makers at various levels such as strategic level, tactical level and operational level in better understanding the performance of the supply chain over a range of parameter settings and to react appropriately. For the strategic level decision maker, it helps in designing the SCN and to negotiate the appropriate contractual/memorandum of understanding issues with other players participating in the SCN. For the tactical decision maker, they help in adjusting/adopting to the situation forced by the external fluctuations without contributing negatively to the entire SCN. This analysis is particularly useful for the operational level managers in making decision with the knowledge of complete SCN behavior immediately. This understanding also helps in a better deployment of lean thinking when designing and managing the lean supply chain. The resource allocation method proposed using cellular neural network is particularly helpful for operational level decision makers as it can solve the issues efficiently and considerably with very good speed.

1.4.3 List of publications


1.5. ORGANIZATION OF THE THESIS


1.5 Organization of the thesis

The section gives an overview of the organization of the thesis and briefly describes the main contents of the succeeding chapters.

In chapter 2, the basic definitions and the concepts of the supply chain are introduced and described in detail. The evolution of supply chain networks from enterprise owned supply chains to vertically disintegrated supply chain networks is detailed in chapter 2. The key components/entities, such as supplier, manufacturer, distributor, retailer and customer, in a supply chain networks are explained in detail in terms of their individual role and their global contribution together with their objectives in the supply chain network. After the explanation of the key components, the key functional issues involved in the supply chains are explained in detail. The major functional issues in the supply chain are also explained in detail by defining each functional issue, the objective of the function and the role of the function in the supply chain network. The objectives of managing the SCN is introduced by defining the supply chain management (SCM). After introducing the definitions of SCM, objectives and principles of the SCM, the key issues involved in managing SCNs’ such as integration, collaboration and synchronization are described in detail. The role of information sharing and information visibility is dealt in detail. The state-of-the-art approaches and methods involved
in managing the SCNs’ efficiently are detailed by considering each key issue. The advantages and limitations/practical obstacles from the state-of-the-art for each of the topics as explained in the literature will be presented and commented.

The uncertainties from the exogenous and indigenous sources play a key role in managing the SCN and makes the supply chain systems complex. In chapter 3, the sources of uncertainties in the SCN are introduced and explained in detail. The effect of uncertainties on the performance of supply chains is illustrated with demand amplification example from literature. The effects of uncertainties on the stability of the SCN and the proposed methods to deal with them from the literature are discussed in detail.

In chapter 4, the objectives of modeling the supply chain are detailed. The major requirements for modeling the SCN are outlined and their detailed description is carried out based on their importance. State-of-the-art modeling methods in SCN are detailed by classifying them into modeling frameworks by standard organizations and simulation based modeling approaches that are developed to analyze and optimize supply chains. Different modeling approaches are compared over a set of comparison criteria, such as ability to quantifying the dynamics, ability to deal with uncertainties, scalability and adaptability of the models, to list their merits and demerits and to show the necessities for nonlinear dynamic systems based modeling and simulation of SCN.

Before proposing the concepts from the nonlinear dynamic systems theory for modeling and simulation of the SCN, the necessary basics to understand the dynamic systems are presented in chapter 5. The linear and nonlinear dynamic systems are introduced for the better understanding of the supply chain model in next chapters. Various possible states of a dynamic system are explained in detail. Bifurcation theory is introduced and how to infer the bifurcation diagrams is presented in chapter 5.

In chapter 6, the importance of identifying/analyzing the nonlinear dynamics in SCNs’ is discussed. A theoretical model of the SCN with manufacturer, distributor and retailer is presented and modeling of the supply chain using coupled oscillatory systems based modeling method is explained in detail in chapter 6. The analysis of the nonlinear dynamics presented in the SCN is illustrated with phase portraits and bifurcation analysis. The basic theory regarding both phase portraits and bifurcation theory is presented in chapter 5. In chapter 7, the basics of nonlinear control and synchronization are introduced. These methods are proposed to control and synchronize the SCN. The popular nonlinear control methods are described before detailing
1.5. ORGANIZATION OF THE THESIS

the method adopted in this thesis. The general overview on synchronization types is discussed in detail and after that the appropriate synchronization types for SCN are discussed in detail. The synchronization method developed to mitigate the nonlinear dynamic effects shown in chapter 6 is explained in detail.

After illustrating the importance of concepts from nonlinear dynamic systems theory on the theoretical SCN model in chapter 6, the usefulness of the concepts on a real-case study in described in chapter 8. The selection criteria for the case study and the background information required to understand the case study is described in detail in chapter 8. Thamagotchi™SCN is considered as a case study to illustrate the importance of analyzing the supply chains with theory and tools from nonlinear dynamic systems. The case study supply chain information is explained in detail from the state-of-the-art. The system dynamics modeling approach is explained in detail in chapter 8. Analysis of the nonlinear dynamics, with bifurcation theory, in Thamagotchi™SCN is presented and the importance of this tool is commented in detail in chapter 8.

The potentials of methods/tools from nonlinear dynamic systems theory in lean SCNs’ is described in chapter 9. Chapter 9 also gives detailed information on importance of lean supply chains. The nonlinear dynamics of the lean supply chain are discussed theoretically in detail in chapter 9. Importance of real time scheduling in lean supply chains is presented in conjunction with importance of dynamic resource allocation in lean SCN. Application of cellular neural networks based analog computing approach for dynamic resource allocation is described in detail in chapter 9. An introduction to cellular neural networks is also presented in chapter 9.

Chapter 10 subsumes the main contributions and goals of thesis with respective to the research questions formulated in chapter 1. It also outlines further work that can be carried out by the knowledge of the methods and tools presented in this thesis.
Chapter 2

Basic concepts of supply chains

2.1 Introduction to supply chain networks

A number of definitions of the supply chain networks have been proposed. Christopher [14] defined it as, a network of connected and interdependent organizations mutually and co-operatively working together to control, manage and improve the flow of material and information from suppliers to end users. According to Jespersen [27], one of the most common perceptions of the supply chain network is, A system whose constituent parts include material suppliers, production facilities, distribution services and customer linked together via the feed-forward flow of materials and the feedback flow of information. Even though there exists many definitions of the supply chain network (SCN), there has been some convergence. The Association for Operations Management (APICS) defines the supply chain network as [28], The global network used to deliver products and services from raw material to end customers through an engineered flow of physical distribution, information, and cash.

Material flow, Information flow, and Cash flow are the three main flows in the supply chain. Figure 2.1 shows bidirectional flow of materials from upstream to downstream, the bidirectional flow of information, and the bidirectional movement of funds. By means of acquisition of raw materials, manufacturing, distribution and customer service, the activities involved in the material flow are delivered to the end-user. The material flow from downstream to upstream include the product returns, recycling and customer requests for repairs and service. Suitable information flows are used to manage all the activities involved in the material flow.
2.2 **Key components of a supply chain**

The global network of supply chains consists of many companies performing many functions in order to deliver the products and services to the end customer. All these companies are the key components of the SCN. In order to understand the SCN better, it is important to understand the basic definitions of these key components.

### 2.2.1 Supplier

Suppliers are those who provide goods or raw materials for the manufacturers to manufacture/assemble a product. Suppliers play a key role in the success of the manufacturer or in other words the success of the product. The quality of the raw materials, the reliability of the suppliers, the ability of the supplier to respond at short notice play a key role in the success of the SCN of the product [29].

### 2.2.2 Manufacturer

Producers or manufacturers are organizations that manufacture/assemble a product. This includes companies that are producers of raw materials and companies that are producers of finished goods. Producers of raw materials are organizations that mine for minerals, drill for oil and gas, and cut timber. It also includes organizations that farm the land, raise animals, or catch seafood. Manufacturers of finished goods use the raw materials and subassemblies made by other producers to create their products. Manufacturers can create products that are
intangible items such as music, entertainment, software, or designs [28]. Producers of tangible, industrial products are migrating to different regions of the world where labor cost is less compared to their previous operating regions.

2.2.3 Distributor

Distributors are companies that take inventory in bulk from manufacturers and deliver a bundle of related product lines to retailers and customers [29]. Distributors also sell products to other businesses in addition to selling larger quantities of products that an individual consumer would usually buy. Thus, distributors are also called as Wholesalers. By means finding and servicing customers by stocking inventory and excessive sales work, distributors typically buffer the producers from fluctuations in product demand.

Products in the inventories that are bought from manufacturers and sold to consumers are owned by distributors. A distributor also performs inventory management, warehouse operations, and product transportation as well as customer support and post-sales service inclusive of product promotion and sales. A distributor can also be an organization that only brokers a product between the manufacturer and the customer and never takes ownership of that product [29], thus only performing the works of product promotion and sales.

2.2.4 Retailer

Retailers stock inventory and sell in smaller quantities to the general public [29]. The tastes, preferences and demands of the customers to whom the products are sold are tracked by the retailers. Retailer, an organization also advertises and offers packages to attract its customers. Combination of price, product selection, service, and convenience are used to attract customers. An example of such kind of combination is price and wide product selection used by discount department stores.

2.2.5 Customer

Customers or consumers are any organization or individual that purchases and uses a product [29]. A customer organization may purchase a product in order to incorporate it into another product that they in turn sell to other customers. Or a customer may be the final end user of
a product who buys the product in order to consume it.

2.3 Key functional issues in a supply chain

The key components of the SCN represent the individual companies participating in the SCN. These companies perform different functions in order to achieve their objectives. Sometimes the same functionality is performed by many companies in a different perspective. In this section we see some of the key functional issues involved in managing the SCN.

2.3.1 Procurement

Traditionally, the main activities of a purchasing manager were to negotiate with the potential suppliers on price and then buy products from the lowest cost supplier. Even though this activity has still got major importance, there are also other activities that are turning out to be equally important. In the recent days, purchasing activity is considered as part of a broader function called procurement [30]. The procurement function can be broken into following main activity categories:

- Purchasing.
- Consumption management.
- Vendor selection.
- Contract negotiation.
- Contract management.

Issuing purchase orders for needed products deals with some routine activities like purchasing activities. Typically a company buys two types of products; 1) direct or strategic materials that are needed to produce the products that the company sells to its customers; and 2) indirect or MRO (maintenance, repair, and operations) products that a company consumes as part of daily operations.

Understanding of what products and how much of each category of products that are bought across the entire company (i.e, by each operating unit) begins the process of effective procurement [30]. This means that there should be a clear understanding of how much of what kinds of products are bought from whom and at what prices. Expected levels of consumption
for different products at the various locations of a company should be set and then compared against actual consumption on a regular basis. When consumption is significantly above or below expectations, this should be brought to the attention of the appropriate parties so possible causes can be investigated and appropriate actions taken.

Contracts have to be negotiated with individual vendors when there is a particular business needs with reference to the preferred vendor list to work out specific items, prices, and service levels. Simple and Complex negotiations are possible. Contracts dealing with purchase of indirect products by selection of suppliers based of lowest price falls under the category of the simplest negotiations. On the other hand, the most complex negotiations are targetted towards contracts to purchase direct materials fulfilling the exact quality requirements and those requiring high service levels and technical support.

2.3.2 Production

Production utilizes the resources to make the final commodity or service that can be delivered to the customer. Production refers to the capacity of a manufacturer to make and store products. Resolving the trade-off between responsiveness and efficiency is the fundamental decision that is faced by managers when making production decisions [30]. Factories and warehouses which are the facilities of production are built with a excess capacities so that they can quickly respond to rapid variations in product demand, while being flexible. Facilities turn to respond lately to the variations in demand when all the capacity (almost all) is being used. On the other hand, capacity costs money and any excess capacity is idle capacity which is not used and which does not generate any revenue. So, the more excess capacity that exists, the less efficient the operation becomes.

2.3.3 Inventory management

Inventory management is a set of techniques that are used to manage the inventory levels within different companies in a supply chain. These set of techniques reduce the cost of inventory to the best possible while still maintaining the service levels required by the customers. Demand forecasts for products and the prices of products provide (act as) the major inputs for inventory management. These kind of inputs for inventory management results in an ongoing process that exploit economies of scale to obtain the best prices for the products and also balances the product inventory levels so that the demand is met. There are four kinds of inventory [31]:
1. Cycle Inventory.

2. Seasonal Inventory.

3. Safety Inventory.

4. Obsolete Inventory.

Inventory that is required to meet product demand over the time period between placing orders for the product is termed as Cycle Inventory [31]. Making fewer orders of large quantities of a product is desirable when compared to continuous orders of small quantities of a product. This kind of making orders results from the economies of scale which in turn makes the Cycle Inventory exist.

Seasonal inventories result from the decision to produce and stockpile products based on the forecast of future demand by a company or a supply chain that has a fixed amount of production capacity [31]. There could be a possibility where future demand might exceed production capacity. In such cases, to meet the high demand in the future, products are produced during low demand periods of time and are stored in the inventories.

Uncertainty existing in a supply chain needs to be compensated and the necessity results in Safety Inventory [31]. On hand safety stock is maintained by the retailers and distributors, so that they don't run out of inventory when unexpected customer demand or unexpected delay in receiving replenishment orders is experienced. So, the higher the level of uncertainty, the higher the level of required safety stock.

Inventory remains in the warehouse, when no sales or usage for a set period of time is experienced, this kind of inventory is termed as Obsolete inventory [31]. Large amount of obsolete inventories result due to the uncertainties in the market.

2.3.4 Distribution

Distribution centers are facilities where bulk shipments of products arrive from single product locations [31]. When suppliers are located a long distance away from customers, the use of a distribution center provides long-distance transportation to bring large amounts of products to a location close to the final customers.
The distribution center may be warehouse inventory for future shipment or it may be used primarily for cross docking. Cross docking is a technique pioneered by Wal-Mart [32] where truckload shipments of single products arrive and are unloaded. At the same time these trucks are being unloaded, their bulk shipments are being broken down into smaller lots and combined with small lots of other products and loaded right back onto other trucks. These trucks then deliver the products to their final locations. Several benefits result from the use of cross docking by distribution centers. A few of them are:

1. Supply chain has faster product flows as only a little inventory (storage) is maintained.
2. Since the product does not need to be put into the storage and then retrieved later, handling expenses are reduced.

However, cross docking being a demanding technique, requires a considerable degree of coordination between inbound and outbound shipments. Capabilities in transporting and delivering goods are aligned closely with the actual needs of the market that are served by the SCN as transporting and delivering is expensive. As customers expect quick delivery in case of highly responsive supply chains, higher are the transport and delivery costs.

### 2.3.5 Transportation management

This refers to the movement of everything from raw material to finished goods between different facilities in a supply chain. In transportation, manifestation of trade-off between responsiveness and efficiency in the choice of transport mode is done. Airplanes that are regarded as fast modes of transportation are very responsive but expensive. On the other hand, ship and rail that are regarded as slower modes are very cost-efficient but less responsive compared to the faster modes. Decisions made in the transportation management are very significant as almost a third of the operating cost of a supply chain is accounted by transportation costs. So, if the value of a product is high (such as a computer), its transport network should have more emphasis on responsiveness and if the value of a product is low (such as grains), its transport network should have more emphasis on efficiency [33].

Transportation Planning Systems help transportation managers in calculating what quantity of materials should be brought to what locations at what times. The systems enable managers to compare different modes of transportation, different routes, and different carriers and also help in creating transportation plans. These systems work with software and data (such as mileage, fuel costs, and shipping tariffs). The former is provided (sold) by system vendors while...
the latter is provided by content vendors [34].

2.3.6 Order management

The process of passing order information from customers back through the supply chain from retailers to distributors to service providers and producers is called Order management. Passing information about order delivery dates, product substitutions, and back orders forward through the supply chain to customers are also inclusive in it. For quite some long time, order management has worked with the use of the telephone and paper documents such as purchase orders, sales orders, change orders, pick tickets, packing lists, and invoices [35].

In the recent decades, supply chains have transformed in a way that they became noticeably as more complex than they previously were. The transformation resulted in multiple tiers of suppliers, outsourced service providers, and distribution channel partners with whom a company deals. Changes in the way of selling products, increase in the expectations of the customer service, and the necessity in quickly responding to new demands in the market has resulted in the complexity of the supply chain. Due to the fact that the movement of data back and forth in the supply chain is quite slow, the lead and lag times are no longer built into the traditional order management process [35]. Necessity of more accurate and faster movement of data exists in complex supply chains in order achieve efficiency and responsiveness while some simple supply chains still work well with the slow movement of data. Techniques that enable more accurate and faster movement of order related data are focussed by modern order management.

2.4 Basics of supply chain management

The term supply chain management was introduced in the early 1980s by Oliver and Webber [36] where they discuss the potential benefits of integrating purchasing, manufacturing, sales and distribution. Houlihan [37] repeats the term to describe the management of materials across organizational borders. Definitions of Supply Chain Management (SCM) have been supplied by several authors. Ellram and Cooper [38] described it as “an integrating philosophy to manage the total flow of a distribution channel from supplier to ultimate customer”. Christopher [14] defined SCM as, the management of upstream and downstream relationships with suppliers and customers to deliver superior customer value at less cost to the supply chain as a whole.
2.5. INFORMATION FLOW IN A SUPPLY CHAIN

The Council of Supply Chain management professionals (CSCMP), has engaged in defining the term supply chain management. The CSCMP definition of the supply chain management (SCM) is following [39]: Supply chain management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third-party service providers, and customers. In essence, Supply chain management integrates supply and demand management within and across companies.

2.5 Information flow in a supply chain

Variety of information flows in the supply chain networks in both upstream from customer and downstream from suppliers and manufacturers as shown in figure 2.1.

2.5.1 Value of information

The design of supply chain should tap the potential availability of more and more information throughout the chain and it should be used for the effective management of supply chain.

• In modern supply chains information replaces inventory [40].

• Information changes the way supply chains can and should be effectively managed, and these changes may lead to, among other things, lower inventories [41].

• By effectively harnessing the information available one can design and operate the supply chain much more effectively and efficiently.

• The supply chain performance can be improved by having accurate information about inventory levels, orders, production, and delivery status throughout the supply chain.

• Using this information effectively does make the design and management of the SCN.

Information availability reduces the variability in the SCN. The information helps suppliers make better forecasts, accounting for promotion and market changes. Information availability also enables the coordination of manufacturing and distribution systems, and strategies. All these benefits together lead to the reduction in lead time and the efficient customer service [42].
2.5.2 Visibility of information

Most data that is gathered today is not real time. Data is gathered using various techniques - manual, barcodes or tags. Different people gather it at different times of the day in individual units of the supply chain, on the shop floor or in the warehouse. This data is then entered in the computerized database at a predetermined time, at regular intervals. Thus, the database is updated in fixed quanta of time. In between these time periods, which typically can last from a few hours to a few days, the database is only pseudo-real time [43]. This introduces visibility gaps in the system.

In many instances, other players in the SCN base their decisions on data that they gather from the player under consideration. They are provided with inaccurate data, which results in sub-optimal decision-making. Achieving the real time visibility is not only related to the technological advancements. The lack of visibility is also due the fact that individual players have different objectives, different infrastructure and different capabilities.

2.5.3 Information sharing

An important strategy for managing SCNs is to share information among SCN partners. One of the main benefits of sharing information is the reduced need for inventory [44]. As a result, the SCN achieves better performance in terms of financial returns, service level, and turn-around times.

Benefits of information sharing

Instead of waiting for the retailer to place orders, the frequency, quantity, and timing of the shipments can be be managed by the manufacturer by using the inventory level information of the retailer if both the manufacturer and retailer share information among themselves [45]. This practice, referred to as continuous replenishment process (CRP), enables the manufacturer to reduce the inventory necessary and to plan the shipments more efficiently.

From the perspective of "bull whip effect", the benefits of information sharing and CRP can be well explained. The "bull whip effect", show that small fluctuation of demands tend to be progressively amplified when moved up the supply chains [42, 46]. Use of safety stock at each stage, varying of batch sizes, frequency of orders, lead-times, and irregular behaviors (forward buying) can be accounted as a few reasons behind the "bull whip effect" in a multi-stage supply
2.5. INFORMATION FLOW IN A SUPPLY CHAIN

Drawbacks of information sharing

Information sharing also has its drawbacks. It may lead to loss of sales if our partners are also having a partnership with our competitors. Sometimes the finer details about the present inventory status can lead to reduction in demand [46, 47].

2.5.4 Practical obstacles for information sharing

Even though information sharing provides numerous benefits and the advancements in the IT sector provides platforms for information sharing, there are also some practical obstacles for information sharing. Interestingly some of the barrier are technical and some of them are social and psychological. The main obstacles for information sharing are [1]:

- Cost and complexity of implementing advanced systems.
- A second barrier is found in systems incompatibility.
- Different Levels of connectivity exist up and down the chain.
- Information is tightly controlled in the absence of trusting relationships.

2.5.5 Role of information technology in supply chains

Supply chain management (SCM) is concerned with the flow of products and information between SCN members and organizations. Several activities to manage the supply chain are coordinated by the recent development in technologies that helps the organization to avail information easily in their premises. As the development of technologies is at a very fast increasing rate, the cost of information is decreased. It is mandatory for everyone making use of the technology to understand that information technology is more than just computers. The functional roles of the IT in managing the SCN are shown in figure 2.2.

Transactional information technology

Transactional IT is concerned with acquiring, processing and communicating raw data about the company’s past and current supply chain operations to its partners in the SCN. Some examples of the transactional IT solutions include [48]:

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Functional Roles of IT in SCM

Figure 2.2: Functional roles of the IT in managing the SCN [1]

- General ledger systems.
- Quarterly sales reports.
- Enterprize Resource Planning (ERP) solutions.
- E-commerce systems.

Transactional IT solutions give the information some sort of reliability than the paper based systems. They also give certain credibility and trust about the accuracy of the data. However, these system only transfer the data what is entered in them. These systems are more useful at the intra-company level to streamline the jobs.

**Analytical information technology**

Analytical IT solutions are concerned with developing and applying systems for evaluating and disseminating decisions based on models constructed from supply chain decision databases. Some examples of the analytical IT solutions include [48]:

- Production scheduling systems.
- Forecasting systems.
- Supply chain network optimization systems.

Analytical IT solutions help in making the decision by either utilizing the information from the transactional IT solutions or by unseing the simulation models of the SCN.
2.6 Integration of a supply chain

In their seminal work, Lawrence and Lorsch [49] defined integration as, the quality of the state of collaboration that exists among departments that are required to achieve unity of effort by the demands of the environment. Even though Lawrence definition restricts itself to internal integration activities of a company this definition can be extended to multiple companies.

Stevens [50] identified four stages of SCN integration. Stage I represented the fragmented operations within the individual company. Stage II focused on limited integration between adjacent functions, e.g. purchasing and materials control. In an individual company, end-to-end planning is internal integration that is required by Stage III while integration of a true supply chain inclusive of upstream to suppliers and downstream to customers is represented by Stage IV.

Global supply chain networks have many players and they are coupled to each other even though the coupling strength varies. To be effective, a supply chain has to link the members of the network and the functions to ensure uninterrupted flow by matching supply and demand flows in a network and securing accurate response at each buyer-seller transaction in the chain. Coordinating these flows in a network requires integration of supply chain partners to ensure unhindered flows at each of the many buyer-supplier interfaces in a supply chain network.

2.6.1 Functional integration

In the multi-echelon supply chain shown in figure 2.1, often gives rise to 'speculative' relationship at each buyer-supplier interface downstream which causes the fluctuation. At each interface, the extent of fluctuation due to speculative relationship gets amplified leading to what is known as "bullwhip" effect [42, 46]. As a result, the supply chain as a whole often carries more inventories than actual requirement and yet there can be pockets where there is not enough. The extent of the fluctuation depend on the coupling strength at the interface between the two players. The coupling strength varies between the key players varies as shown in figure 2.3. The coupling strength downstream is very weak as the players are loosely coupled in their relations. The relations between the downstream players is weak as we can see in figure 2.3, this is mainly due to the long standing relationships governed by the contracts and trust. Advances in IT solutions are one of the key enablers of the functional integration across the SCN.
2.6.2 Practical obstacles for integration

The major obstacles for the integration are lack of trust, different goals and priorities and lack of parallel communication structure between the partners in the SCN. The operational tool obstacles (manual performance data management and non-standardized performance metrics) were seen on an overall level to hinder SCN process integration [51].

2.7 Collaboration in a supply chain

Collaboration is a recursive process where two or more people or organizations work together in an intersection of common goals [52]. Joint planning, coordination, process integration between suppliers, customers, and other partners in a supply chain is focussed by collaboration which is a recent trend in supply chain management (SCM). Reduction of costs, increase in the return on assets, and increase in reliability and responsiveness to market needs are some of the competitive benefits of collaboration. Collaborative SCM goes beyond mere exchanging and integrating information between suppliers and their customers, and involves tactical joint decision making among the partners in the areas of collaborative planning, forecasting, distribution, and product design. The result of collaborative SCM is not only the reduction of waste in the supply chain, but increased responsiveness, customer satisfaction, and competitiveness among all members of the partnership. Thus, collaborative SCM systems allow organizations to progress beyond mere operational-level information exchange and optimization and can transform a business and its partners into more competitive organizations.

Collaboration is a very broad and encompassing term and when it is put in the context of the supply chain it needs yet further clarification. Many authors when talking about collaboration cite mutuality of benefit, rewards and risk sharing together with the exchange of information as the foundation of the collaboration. Transition to the collaboration from the negotiation
phase is shown in figure 2.4. True collaboration partnerships are based on high levels of trust, commitment and information sharing among the partners [52, 53]. Partners throughout the supply chain must be integrated into others’ processes. Staff need to accept that a company, although perhaps playing a comparatively minor role in the supply chain, has relations with many partners, and that its business decisions can have a significant impact on their own performance as well as that of the whole supply chain. Close collaboration relationships with partners; including manufactures, suppliers, distributors, transporters and end-customers are the key to success. Therefore, companies must collaborate with partners towards common goals and mutual benefit, as well as for the benefit of the individual company. Inefficiencies, excess stock, slow response and lost profits would result from distortion of information caused due to a collaboration failure [2]. There are many elements of collaboration that have been identified in the various literatures in and around supply chain management. One of the major supporting elements of collaboration is a "collaborative" culture (see figure 2.5, which is made up of a number of elements: trust, mutuality, information exchange, and openness and communication.

2.7.1 Collaborative Planning, Forecasting and Replenishment (CPFR)

Voluntary Inter-industry Commerce Standards (VICS) association, a retailer trade association, responsible for promoting common standards and business processes used to the continuous pursuit of improving both the customer satisfaction and the efficiency of business trade relationships, proposed the CPFR system [54]. CPFR, as its name implies, responds to the conduct of an enterprize’s planning, forecasting and replenishment by drawing its supplying products with collaborative operations of all trading partners through the distributed channels in SCN. CPFR originated in 1995 as an initiative co-led by Wal-Mart and consulting firm Benchmarking Partners. This initiative originally was called Collaborative Forecasting and Replenishment. VICS created guidelines for CPFR in 1998. Since the development and publication of these guidelines, over 300 companies have successfully implemented CPFR [54]. The implementa-
tion of CPFR has also extended to industry sectors beyond retail, including high-tech industries.

The CPFR model offers a general framework by which a buyer and seller can use collaborative planning, forecasting, and replenishing processes in order to meet customer demand. Though four collaboration activities, in which the buyer and seller are involved are listed in logical order to increase performance, simultaneously involvement in these activities is often performed by the companies. Understanding the relationship between the buyer and seller by the same and establishing product and event plans forms the first collaboration activity which is called Strategy and Planning. Forecasting of customer demand and shipping requirements forms the second collaboration activity called Demand and Supply Management. The third collaboration activity involves placing, receiving, and paying for orders, and also preparing, delivering, and recording sales on shipments and it is called Execution. Analysis is the fourth and final collaboration activity that is targeted to work towards continuous improvement which is done by monitoring the third collaboration activity (execution step) and measuring the key performance metrics. Further details on implementing the CPFR initiatives can be obtained from the VICS whitepaper on CPFR implementation [54].

Figure 2.5: Key entities in the collaboration of SCN [3]
2.7.2 Practical obstacles for collaboration

It is suggested that many of the problems related to supply chain collaboration are due to a lack of understanding of what collaboration actually implies. Skjoett-Larsen [3] found that a major barrier to the development of CPFR (collaborative planning, forecasting and replenishment) initiatives was a lack of attention to developing front end agreements as to specifically what organizations were going to collaborate over. This poor understanding is further increased due to the association of collaboration with the hype surrounding e-business whereby technology has been promoted as the key to enabling wide-scale inter-organizational collaboration.

Another major barrier would appear to be the context for collaboration, in terms of when to collaborate and with whom. Some of the confusion surrounding this issue would appear to come from a number of sources, including the implication that collaboration must be scalable to a large number of customers and suppliers [3]. This in itself is not a major barrier, but it does serve to confuse organizations in terms of the value that may be derived from collaboration. SCN collaboration requires the commitment of significant resources to implement it, and organizations that try to collaborate with a large number of their customers and suppliers will not succeed. The cost of such wide-scale implementation would simply outweigh the value derived from such an effort.

Many of the elements that have influence on collaboration as shown in figure 2.5, such as culture, trust, information exchange and supply chain wide performance measures have been to a large extent ignored due to their complexity, and deserve significant attention individually in terms of further research. Further research is also required to develop a deeper understanding of the relationships between these elements of collaboration.

2.8 Synchronization of a supply chain

In this section we briefly introduce the importance of synchronization and present the modern methods/tools used to deal with it. When organizations synchronize supply chains and have real-time access to data, they enjoy several competitive advantages:

- Anomalies and exceptions are identified early and the data for intelligent response is immediately available. This greatly minimizes the bullwhip effect and saves downstream partners and customers from needless activity.
• Back-end processing of change orders, modified invoices, and updates to inventory systems can happen as decisions are being made [55].

• Visibility into partner systems makes planning easier and enables managers to see opportunities that were not apparent before. This visibility also enables supply chain partners to collaborate more effectively. This cooperation is essential when problem resolution requires the coordination of several supply chain partners.

• Easier integration and expansion. Since the synchronization and data passing mechanisms are not based on packages from supply chain software vendors or ERP firms, the software is optimized for all players. Packages can be changed without disrupting the infrastructure and new partners can be added easily.

2.8.1 Definition of synchronization in supply chains

Jeff Ashcroft, VP of Logistics/Supply Chain at PricewaterhouseCoopers LLP, specifically defines the SCN synchronization as, *a tight coordination of current data, historical data, transaction, physical processes, and activity schedules of all participants.*

To date, the majority of supply chain efficiencies has come from improvements within the four walls of each individual company [56]. Certainly, there has been collaboration between trading partners in terms of improved communication (EDI and current internet based web information exchanges), better information (point of sale data and the CPFR initiatives), and a general willingness to work more closely together. Even though these information sharing methodologies have provided synchronization to some extent, still there exists some discrepancies. Apart from all these data discrepancies, there may be other disturbances which occur due to external perturbations. There are different methodologies applied to study these discrepancies and to quantify their effects. In this thesis we focus on two major synchronization types applied in supply chain networks based on the supply chain model [55] namely:

• Complete Synchronization: the cycle time at an upstream stage equal to cycle time of the next downstream stage.

• Partial synchronization: the cycle time at an upstream stage is an integer multiple of the cycle time of the next downstream stage.

2.8.2 Limitations of the synchronization tools

Supply chain synchronization with the help of the IT solutions is only possible if the integration and collaboration between the partners is completely realized as shown in figure 2.6. As we
Figure 2.6: Stages towards achieving synchronization of SCN

have seen in previous sections of this chapter, the realization of the complete integration and cooperation depends on the information visibility, information availability and the aptness of information sharing. These constraints limit the ability of the synchronization tools.

2.8.3 Research avenues for synchronization in supply chains

SCN synchronization provides numerous research avenues as it includes many complex issues involving the physical processes, coordination of data and functions, and scheduling of activity of all the participants. The modern transaction based IT solutions are partly successful in synchronizing the available data. But these solutions are of limited help if the data discrepancies occur or the data becomes irrelevant due to market fluctuations. This thesis particular focuses on synchronizing the SCN when it is subjected to external fluctuations.

2.9 Summary

This chapter introduces definition of the supply chain network from the perspective of different professional organizations and professionals from different fields. The key components/entities, such as supplier, manufacturer, distributor, retailer and customer, in a supply chain networks
are explained in detail in terms of their individual role and their global contribution together with their objectives in the supply chain network. After the explanation of the key components, the key functional issues involved in the supply chains are explained in detail. The major functional issues in the supply chain are also explained in detail by defining each functional issue, the objective of the function and the role of the function in the supply chain network. The objectives of managing the SCN is introduced by defining the supply chain management (SCM). After introducing the definitions of SCM, objectives and principles of the SCM, the key issues involved in managing SCNs’ such as integration, collaboration and synchronization are described in detail. The role of information sharing and information visibility is dealt in detail. The state-of-the-art approaches and methods involved in managing the SCNs’ efficiently are detailed by considering each key issue. The advantages and limitations/practical obstacles from the state-of-the-art for each of the topics as explained in the literature are presented.
Chapter 3

Uncertainties in supply chains

3.1 Overview of uncertainties in supply chains

Even though integration, collaboration and synchronization of the stakeholders supported by advanced IT infrastructure should provide the ideal solution for the optimization of the supply chain networks, in many cases it is not [57, 22, 58]. Business is not usual always; there are many things that happen with time. The major source for uncertainties in the SCNs’ originates from the fact that operations are performed over long period of time and stakeholders are dispersed globally. SCM is a complex set of processes and flows (information flow, product flow, monetary flow) that can either add a great deal of efficiency and profit or can cause serious problems for a company. SCM is technology intensive in some aspects, in some ways it can be compared to a simple gossip game. In the game, one passes the secret to his friend, which is then passed around a circle of friends. The secret can be true or false, but what is usually the case with the secret is that it becomes exaggerated - if the secret starts out as ”there is an insect outside,” it ends as ”a monster is in the school”. The supply chain can often be seen as the same circle.

Today’s supply chains pass information relating to item information, forecasting and replenishment upstream and downstream among manufacturers, brokers, distributors, retailers and warehouse, transportation, management and global trade systems. This results in a flow of information from a point at the manufacturer where the product manufactured to a point at the end consumer where the product is being sold. Quality of the data at the start point of the supply chain often influences the accuracy of information and thus the same information needs to be maintained throughout the supply chain. If the information is wrong in the beginning, it never has a chance of being correct when it finally gets up or down the supply chain. In fact, if inconsistencies remain, there are chances for errors to propagate through the whole supply chain.
The definition of a truly efficient SCN is when all companies involved are communicating correct or reliable data - a manufacturer is communicating the reliable product information and receiving accurate purchase orders; a retailer is receiving the specific products that were ordered; and the product is available to the end consumer at the right time and at the right price. However, communicating the reliable data is always not possible in SCNs’ as each stake holder have different objectives and constraints. The traditional SCM has been based on limited information sharing which is restricted to the product in consideration and transaction-oriented towards that product [29]. Apart from distorted information, there are many other sources that are contributing to the generation of uncertainties at various stages in the SCN. These sources are explained in detail in next section.

### 3.2 Sources for uncertainties

The entities in modern SCN are globally distributed from pacific to Atlantic. This global nature brings uncertainties in the SCN at many levels and in many forms such as transportation, lack of visibility, socio-cultural differences. In this thesis, the sources of uncertainties are mainly classified into two, namely;

- Internal uncertainties - Uncertainty generated/caused by the entities participating in the SCN.
- External uncertainties - Uncertainty generated/caused by the external factors on which the participating SCN entities have no control.

#### 3.2.1 Internal uncertainties

The globalization of SCN operations also introduced the vertical disintegration of organizations as a measure for efficiency and cost cutting. Vertical disintegration in effect increases the length of SCN i.e. more independent entities. These independent entities generate/cause lot of uncertainties with their handling/managing of activities pertaining to them as shown in figure 3.1.

**Supplier**

In recent years, manufacturing companies are depending upon many suppliers for their requirements such as raw material and semi finished goods. Manufacturing companies are increasingly
3.2. SOURCES FOR UNCERTAINTIES

![Diagram showing sources of internal uncertainties in SCN]

Figure 3.1: Sources of internal uncertainties in SCN

becoming assembly systems, i.e. outsource all the components and assemble them as per the customer’s need. This process indirectly increases the length of the supply chain upstream from the manufacturer. The effect of uncertainty is more when the manufacturers depend solely on a unique supplier for a particular material. The other complexity added to this is the suppliers are again distributed geographically all over the world. Some of the uncertainties that can be generated from the suppliers are listed as below [24]:

- Dispatching wrong material - Many suppliers supply similar products with very small changes to many manufacturers. Some times this will contribute to dispatching the wrong material to a manufacturer.

- Lot with poor quality products - Poor quality control at supplier effects the manufacturer as it delays the processes to wait for the quality product.

- Delay in sourcing the material.

- Reliability and financial stability of supplier.

The effects caused by the uncertainty at the supplier level snow ball into the down stream. Manufacturing companies are doing more analysis, both formally and informally, to cope up with the supplier level uncertainties. Manufacturers are increasingly looking towards large suppliers with reputation and financial stability.
Transportation

Global SCNs’ heavily depend on reliable and secure transportation networks. The transportation link is one of major source for interactivities within the SCN as the operations are spread in wide geographical area and with different modes of transportation. The multi-model nature of transport introduces different type of uncertainties depending upon many external factors such as environmental conditions and geo-political issues. In order to handle this complex issue of transportation management, companies are relying on the third party logistics service providers (3PL). Some large sized manufacturers depend on the fourth party logistics service providers (4PL), who in turn manage many 3PLs, to manage their transportation requirements from the suppliers and to the distributors. These processes again increase the length of the SCN and the sources of uncertainties. The uncertainties manifested within the transportation of material/products directly reflects at the customer end with delays. Some of the sources of uncertainties at the transportation link in the SCN include [24, 23, 59]:

- Accidents - Traffic accidents involving trucks carrying the product have direct effect on the delay and in some cases the accidents also delay the trucks even if they are not involved in the accident by means of road blockades.

- Load consolidation, resource allocation, scheduling - The consolidation of loads by the transportation companies, allocation of resources and scheduling of the vehicles all have direct effect on both upstream and downstream in terms of delays, quality and reliability.

- Security of carriers - Terror strikes, pirates and the preventive random security checks contribute to the delay during delivery of goods.

- Reliability of 3PL - As many of the transportation activities are outsourced to service providers the reliability and ability of the service providers have direct effect on the delays.

- Co-ordination between 3PL and 4PL - As 4PL service providers manage many 3PLs, the reliability between 3PL and 4PL is also important for the manufacturer even though he is not part of the relationship between them.

Companies are looking at advanced transportation management solutions (TMS) as a tool for the better management of the transportation needs. But many companies are looking towards reliable 4PL and 3PL service providers as means to avoid the risks associated with transportation.
3.2. SOURCES FOR UNCERTAINTIES

**Manufacturer**

Manufacturer is the heart of the SCN of the product. Manufacturer holds major responsibility, in many cases, for the product to reach the customer without much delay. Manufacturer responds quickly to any uncertainties with him and also to the effects of uncertainties both upstream and down stream. Even though he is not mainly responsible for many uncertainties, he is the major player to receive the effects of uncertainties. In responding to uncertainties, manufacturer without his knowledge contributes to the generation of uncertainties. Some of the sources for uncertainties at manufacturer are listed below [22]:

- Machine break down - Machine break downs are unexpected and they cause the uncertainties in assembly line and uncertainty down the SCN.

- Production schedules, order batching - The uncertainties coming from the basic production operation or invisible and they also have snow balling effect on the SCN.

- Personnel availability, strikes, schedules - Even though majority of the manufacturing is automated, still there is an important role played by the personnel in manufacturing plants. The issues related to personnel such as availability of the required personnel in required time shift, personnel strikes as part of union also some times create the non desirable effects in SCN.

- Safety stock management - Safety stock management policies generate long term uncertainties especially down stream in long run, if not applied properly.

**Distributor**

Distributors or wholesale distributors are the bridge between the manufacturer and retailers. Distributors generate uncertainty both upstream and downstream. The transportation activity at the distribution center is one of the key contributors of uncertainty to the retailers. Apart from transportation activity, distributors also generate uncertainty due to the following issues [22]:

- Resource allocation schemes - Resource allocation policies followed by distribution managers, dispatcher, to different retailers may be ill-defined causing uncertainty to retailers.

- Skill levels of dispatcher - The varying skill levels of dispatcher has direct influence when he is responding to the uncertain situation.

- Inventory management policy - Inventory management policy depending upon the capabilities of the distributor can cause uncertainty to manufacturer.
Retailer

Retailer is the prime responsible person in understanding the demand generated by the customer. The inputs given by the retailer are very important for upstream managers in forecasting the demand. But the retailer is very loosely connected with the upstream. The retailer generates uncertainty mainly from the following issues [59]:

- Poorly designed contracts with distributor.
- Dealing with products from competitors.
- Communication network with upstream.
- Ordering pattern with the distributor.
- Ratio of demand satisfaction by distributor.

3.2.2 External uncertainties

Apart from the uncertainties manifested from the sources within the SCN, the entities in the SCN are more affected by uncertainties from external sources. Internal uncertainties can be managed to a certain extent by the well drafted contracts. The external uncertainties are hard to predict and in most cases we can’t predict that some anomaly is crept into the SCN till the effect is felt by all the players in the SCN [22].

Visibility

The increased length of SCNs is increasing the time taken for material to flow from one end to the other end of the supply chain. The increased outsourcing of the supply chain process offshore and sub-contracting many of the manufacturing activities are again contributing towards the increase in the time it takes for the product to reach the customer. This increased length of number of players and the time for the end-to-end activity decreases the visibility of the processes between the players. Hence, it is often the case that one entity of a SCN has no detailed knowledge of what goes on in other parts of the SCN – e.g. finished goods inventory (FGI), material inventory, work-in-process (WIP), actual demands and forecasts, production plans, capacity, and order status [60]. This lack of knowledge about the processes and operations within the SCN leads to decision taking without the global knowledge thus generating uncertainties both upstream and downstream. Dvorak [4] highlights the importance of visibility through the figure 3.2, detailing the processes involved in the DVD player SCN. Dvorak also stresses the why it is so difficult for each participant in a supply chain to gain accurate demand visibility.
3.2. SOURCES FOR UNCERTAINTIES

Figure 3.2: Visibility issues in a global technical devices SCN (taken and adapted from [4])

**Market fluctuations**

Capgemini Consulting in their recent survey of 365 SCN managers around the world about the impact of recession and market fluctuation on the SCM [61], 65% respondents said the market fluctuations affects their decisions at various stages and indirectly affects the SCN in long run. About 54% respondents said that inventory reduction and cost cutting measures as their preferred strategy for adapting to market fluctuations. The uncertainties introduced by market fluctuations also have long term effects on the relationships between the players in the SCN. Market fluctuations also introduce forecasting fluctuations. Forecasting errors in terms of demand prediction have greater impact on the performance of supply chains [62]. Fluctuations in demand forecasting induces uncertainties at retailers and these uncertainties snow balls upwards in the SCN.

**Social and psychological issues**

The physical distance between the geographically distributed entities of the SCN is reduced greatly with efficient transportation systems and communication systems. However, the psychological distance between the entities due to socio-political issues remain the same. This is a major cause of concern in global supply chains where trust and confidence of all the players in the SCN is important for the performance of SCN as a whole [60]. The mis-
trust, over reactions between the players can create an uncertain environment for all the entities.

### 3.3 Demand amplification due to uncertainties

Demand amplification is a situation whereby small variation in demand by customers result in increasingly large variations as demand is transmitted upstream along the supply chain as shown in figure 3.3. Inventory is generally used as insurance against the uncertainties. In the case of a single enterprise-based SCN, it is relatively easy to overcome the uncertainties with properly sized inventories at each stage such as raw materials, work in process and finished goods inventories. The present day statistical tools and forecasting methods can satisfactorily aid in determining how much must be held to satisfy the customer demand for the particular product despite the uncertainties [21]. However, the problem is much more complicated when considering the whole network consisting of different players distributed globally. Practically each player holds some inventory to protect against uncertainties, but the real difficulty is in determining how much must be held and where to hold it. To date, there is no clear analytical way to calculate the propagation of uncertainties up and/or down the supply chain.

![Figure 3.3: Amplification of demand due to uncertainties in SCN](image_url)

Chen [63] demonstrated that a base stock policy, companies place orders upstream equal to those they receive from downstream, minimizes the total supply chain cost. Chen also demonstrated base stock policy avoids the demand amplification when the demand distribution
3.4 Effects of uncertainties on the stability of supply chain

Traditionally, companies have relied on the experience and intuition in facing uncertainties. The decisions made with intuition can make the SCN exhibit various dynamic states [22]. The local decisions made by the concerned managers to satisfy a particular effect caused due to uncertainty may effect other entity at either end of the SCN. The propagation of these effects in both up and down the stream can also lead to chaotic state. Chaos is defined as an aperiodic, unpredictable and bounded dynamic in a deterministic system with sensitivity dependence on initial conditions [65]. Chaos is a disorderly long-term evolution occurring in deterministic non-linear systems.

The beer game developed at MIT to introduce students and industrialists to the concepts of economic dynamics shed further light on supply chain dynamics [25]. The game is played by four teams of participants; each representing an entity within the supply chain i.e. a retailer, wholesaler, distributor and the factory. This beer game is a good case to study the effects of the inter-relating feedback loops within the supply chain. The game shows how the SCN give rise to complex behavior within what seems to be a very simple business system. The researchers at MIT investigating managerial decision-making behavior found that participants apply simple rules for making ordering decisions when playing the game [3]. It has been found after many runs of the beer game, some participants perform better by using their experience. The experienced players consider the inventory along the line while other ignore it all together or forget occasionally. It has been found that one in four management teams in the supply chain creates deterministic chaos in the ordering pattern and inventory levels [22]. The results
also showed that the slightest change in policy could result in a stable output flipping into the chaotic region, i.e. an error when inputting an order, an order delayed in the autocratic process or post, a player forgetting something. Dramatic and costly effect on the management of the supply chain can be caused by all these everyday seemingly inconsequential delays or errors. As a result of this, increase of considerable sub-optimal costs to the system is evident that and these can be exceeding over 500 per cent of the minimum possible costs [22]. Many other researchers also used beer distribution game to study empirically the factors influencing the SCN stability. The major finding for the cause of instability of the SCN is broadly related to the poor reaction of the players to the supply line of orders placed but not received.

The beer game simulations only showed how and why supply chains exhibit various non-stable states under the influence of internal uncertainties, fluctuations caused by uncertainties even when players are experienced and highly educated. But in real world scenarios, the external uncertainties have more negative influences on the whole supply chain and the decisions taken by the demand managers are influenced by market information and managers understanding of external fluctuations. In some cases, managers can’t detect the effect of the uncertainties till the chaos occurs. This thesis mainly focuses on analyzing the effects of external uncertainties and mitigating the effects caused by them.

3.5 Summary

The sources of uncertainties in the SCN are introduced and explained in detail in this chapter. The uncertainties from the exogenous and indigenous sources play a key role in managing the SCN and makes the supply chain systems complex. The effect of uncertainties on the performance of supply chains is illustrated with demand amplification example from literature namely beer distribution game developed at MIT. The effect of both exogenous and indigenous uncertainty sources on the demand amplification is discussed in detail separately. The effects of uncertainties on the stability of the SCN and the proposed methods to deal with them from the literature are discussed in detail. The beer game simulations only showed how and why supply chains exhibit various non-stable states under the influence of internal uncertainties, fluctuations caused by uncertainties even when players are experienced and highly educated. However, the influence of external uncertainties on the performance of the players in the game and the supply chain as a whole is not considered in the beer distribution game. The major finding for the cause of instability of the SCN is broadly related to the poor reaction of the players to the supply line of orders placed but not received.
Chapter 4

Traditional modeling approaches in supply chain networks

4.1 Objectives of modeling in supply chain networks

A model is defined as “a representation of a system for the purpose of studying the system” [66]. A model is a simplification of a system, but contains those components that are identified as relevant to the problem under investigation. Models are used to gain insight or predict future performance of a system. Modeling refers to the process of creating a representation of a system.

In the past, many companies have focused on improving their productivity and efficiency within their company, such as through computer-integrated manufacturing systems (CIMS), inventory management systems (IMS), transportation management system (TMS) and factory automation (FA). However, these activities have reached the limit of improving profits in varying customer needs. SCN encompasses the many integrated processes such as procurement, production, and distribution by which raw materials are converted into finished products and delivered to customers. Because of the individual objectives of the players in the SCN, differences in the business environments and market requirements, the SCN should be modeled to meet the specific global goal of the SCN. Modeling can assist in the design and implementation of a new supply chain and efficiently managing the already existing supply chain. According to Vernadat [12], there are two basic aspects in supply chain modeling:

- The SCN should be modeled in order to manage it properly.
- The processes to be integrated and coordinated need to be modeled.

Therefore, the model should be able to capture the complexities of the SCN and facilitate the
Li et al [67] summarized the main motivations for supply chain modeling:

- Capturing supply chain complexities by better understanding and uniform representation of the supply chain.
- Designing the supply chain management process to manage supply chain interdependencies.
- Establishing the vision to be shared by supply chain partners, and provide the basis for internet-enabled supply chain coordination and integration.
- Reducing supply chain dynamics at supply chain design phases

### 4.2 Requirements for modeling a supply chain network

The complex nature of the SCN management makes the modeling of the SCN dynamics a complex issue. The availability of the data from different players and from different stages is one of the crucial aspect for an efficient decision model. In SCN management decisions are taken at several stages usually classified into three main categories namely [29]:

- Strategic decisions related to the global supply chain design with involvement of the most players in the SCN. The decisions taken at the strategic level are either taken during the supply chain design or re-design stage.
- Tactical decisions are midterm in nature. These decisions are taken to improve the performance of the SCN by considering the modifications/changes needed by observing the performance of the SCN.
- Operational decisions are related to the day-to-day activities. The operational decisions are taken at the individual level in the SCN.

In order to model the SCN, that is inclusive of all the dynamics at the these three stages it is important to address the following modeling issues [5]:

- Problem identification: Most of the time it is not reasonable to model every details of the SCN. It is a good idea to focus on the problem areas.
- Environmental analysis: Understanding the business process and industry characteristics is important to model the complete system dynamics.
Variable identification: Identification of the crucial variables which have high influence on performance metrics is important especially for decision makers at the tactical and operational level.

Model categories or selection: Each SCN is unique and the purpose of the model also varies depending upon the product. Studying and selecting the suitable model category is important for efficient use of the model.

Multiple model use: Each modeling type has its own advantages and disadvantages. In some cases it is also useful to consider the multiple models for efficient SCM.

4.3 Classification of modeling approaches in supply chain

There are a number of supply chain modeling methods that have been proposed. The supply chain modeling is mainly classified into two categories [5, 12]. The first categories deals with the modeling frameworks from the standard organizations or the group of companies. The modeling frameworks are mainly targeted for the strategic decision makers. These frameworks provide some sort of standardized approaches in designing the SCN. The other category involves the simulation models of different types. These simulation models can be again classified into several categories as shown in figure 4.1.

![Classification of supply chain model categories](image)

Figure 4.1: Classification of supply chain model categories [5]

Deterministic models assume that all the variables are known and can be specified with certainty, whilst stochastic models have at least one variable that is unknown and assumed to follow a particular probability distribution [5, 12]. IT-driven models reflect the proliferation of IT applications for supply chain modeling through rapid developments in Information Technology. These models target integration and coordination of various activities based on real-time application throughout the supply chain, including a variety of different systems and system modules, such as warehousing management systems (WMS), enterprize resource planning (ERP), geographic information systems (GIS), and aspects of various forecasting,
CHAPTER 4. TRADITIONAL MODELING APPROACHES IN SUPPLY CHAIN NETWORKS

<table>
<thead>
<tr>
<th>Level</th>
<th>Modeling Method</th>
<th>Model Detail</th>
<th>Model Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operational level</td>
<td>Mathematical model</td>
<td>Very detailed</td>
<td>Small</td>
</tr>
<tr>
<td>Tactical level</td>
<td>Optimization/discrete event simulation</td>
<td>Normal</td>
<td>Normal</td>
</tr>
<tr>
<td>Strategic level</td>
<td>Combined modeling simulation</td>
<td>Simple</td>
<td>Very large</td>
</tr>
</tbody>
</table>

Table 4.1: Appropriate modeling methodologies based on the purpose of the model [12]

distribution and transportation systems. The simulation based models also different depending upon the application, i.e. the purpose of the model as shown in table 4.1.

4.4 Supply chain modeling frameworks

The supply chain modeling frameworks helps in identifying the key aspects in designing the SCNs. The frameworks provide the reference models to compare with and helps in benchmarking the best modeling practices. However, the acceptability of these reference frameworks depends on the success of the reference models and the organizations behind them. The two popular modeling frameworks in this direction are proposed by the Supply-Chain Council (SCC) and Global Supply Chain Forum.

4.4.1 SCOR (Supply Chain Operations Reference) model

The Supply Chain Operations Reference-model (SCOR) has been developed and endorsed by the Supply-Chain Council (SCC). It is an independent not-for-profit corporation formed in 1996, with an initiative to develop a supply chain implementation model. This model is a business process reference model, which is designed for effective communication among supply-chain partners and to provide a comprehensive set of tools linking business processes to metrics, best practice and technology [6]. SCOR integrates elements of benchmarking, leading practices, and business process engineering into a single framework, with SCOR, supply chain management is defined as the integrated processes of PLAN, SOURCE, MAKE, DELIVER, and RETURN, from the supplier’s supplier to the customer’s customer. Figure 4.2, details the frame work of SCOR showing all the integrated processes. The experiences from the industry in using the SCOR model and the finer details of the SCOR model can be accessed by public at SCC Web site [6].
4.4. SUPPLY CHAIN MODELING FRAMEWORKS

4.4.2 Global Supply Chain Forum (GSCF)

The Global Supply Chain Forum (GSCF) was directed by Dr. Douglas M. Lambert. Independent of specific functional expertise, critical issues relating to operational excellence and customer satisfaction are provided to the leading practitioners and academics for pursuing. The Global Supply Chain Forum (GSCF) has introduced another Supply Chain Model like SCOR. Eight key business processes, both cross-functional and cross-firm in nature forms this framework [68]. Each process is managed by a cross-functional team, including representatives from logistics, production, purchasing, finance, marketing and research and development. The critical linkages in the supply chain are formed from the the relationship management processes of customer and supplier while the key customers and suppliers are interfaced by each process. GSCF offers the following services [68]:

- Provides the platform to share information among Forum members without competitors present.
- Provides the opportunity to develop relationships with peers in other industries.
- Conducts research to install leading edge practice in Forum organizations. Identify implementable approaches, useful tools and measures.
- Make research available through publications and other means.
- Provides opportunities for constructive feedback to improve business practices.
- Conducts on-site benchmarking and other benchmarking opportunities.
- Solves problems using depth of experience of group.
- Provides diverse perspectives of the global supply chain.

Figure 4.2: Supply Chain Operations Reference model framework [6]
4.5 Simulation based modeling approaches and examples of related simulation tools

Simulation is the process of designing and creating a model of a real or proposed system, using abstract objects in an effort to replicate the behavior of their real-world equivalents [69]. Under some given conditions, the behavior of the system is shown by the changes in the parameters of a model over a period of time with the model being dynamic [69]. Simulation is considered as one of the most powerful techniques to apply within a supply chain environment. Wyland et al [70] debates that the strengths of the simulation techniques in evaluating interdependencies and system variation are increasing its popularity as a tool in SCM. This enables a decision-maker to assess changes in part of the supply chain and visualize the impact of those changes on the other parts of the system, and ultimately on the performance of the entire supply chain.

4.5.1 Systems theory based modeling

The system dynamics based approach to model the business dynamics was first introduced by Forrester [7]. System dynamics is a computer-aided approach for analyzing and solving complex problems with a focus on policy analysis and design. Initially called Industrial Dynamics [7], the field developed from the work of Jay W. Forrester at the Massachusetts Institute of Technology. Using the Forrester Model shown in figure 4.3 as an example, Forrester (1961) describes the modeling process used in modeling continuous processes, whilst clearly emphasizing the importance of information feedback to the System Dynamics method. The stages of model conceptualization, model parameterizations, and model testing through various experiment is illustrated by him by referencing the problem identification and the formulation of questions to be answered as the first step in a system dynamics study. Forrester also differed from the operations research experts in dealing the problem company wide. Forrester identified the future of the industries lies in collectively solving the problems.

System dynamics understands the dynamic behavior of complex physical and social systems using an approach based on perspection of delays and information feedback as it has its origins in management and control engineering. System dynamics is an approach which is actively used to model the managerial behavior. In 1989, [25] proposed a model that can be used to analyze the supply chains using the industrial dynamics fundamentals proposed by Forrester [7]. Ster-
4.5. SIMULATION BASED MODELING APPROACHES AND EXAMPLES OF RELATED SIMULATION TOOLS

A man uses the beer game ([25, 71] to conduct an experiment on managing a simulated industrial production and distribution system. The Beer game presents a multi-echelon production distribution system, containing multiple actors, non-linearities, feedbacks and time delays throughout the supply line. The players are advised to minimize costs by managing their inventories under uncertain demand and unknown delivery lags. Oscillations are exhibited by the system during the course of a simulation run. Long time lags that exist between placing an order and receiving the goods are not taken into account by the decision rules that are applied. Supply chain dynamics are analyzed using the Sterman model.

Figure 4.3: Forrester’s industrial dynamics model [7]
Due to its visual nature and simplicity, after a lengthy floppy period, the system dynamics approach is gaining momentum in modeling the inventory management processes, in policy development and demand amplification [72]. The system theory based modeling is also used to develop the feedback controllers to mitigate the bullwhip effect (i.e. the demand amplification) to some extent [73].

### 4.5.2 Discrete event modeling

In order to achieve a competitive advantage, supply chains are continually faced with challenges to improve their processes and to adapt to customer demand. Effective supply chains are those designed to deliver products and services promptly and reliably, at low cost and with high quality. Fluctuations in demand and production change dynamically, which makes supply chains complicated to understand. Many analytical models only take a few variables into consideration. This simplification could result in a model that is not a realistic representation of the system. Simulating a supply chain is complex because a model must include several processes. However, to ignore some processes could lead the simulation model to ignore critical functions and activities that affect the performance being measured [74]. To define the processes involved and develop a model that provides the user with a holistic view of the supply chain, thus preventing sub-optimization, is thus a fundamental step in simulating supply chains. Using simulation will allow the user to understand the implication of variances on the supply chain, like labor variance, material variance, and most importantly, demand variance. Using deterministic values in simulation models can result in response values far from the real system responses. In discrete event simulation, the variables change at discrete sets of points in time.

To overcome the modeling insufficiency by several assumptions, simulation is widely used to evaluate the impact on supply chain performance as a decision supporting tool for the supply chain optimization. In particular, discrete event simulation is emerging as the most suitable tool for the evaluation of the supply chain performance due to the powerful and realistic modeling and analysis characteristic [8]. However, the complexity of supply chain optimization problem and the stochastic nature of DES cause the unaffordable computational load; the evaluation of a large number of alternatives for supply chain optimization is in a class of NP-hard problem and the number of simulation replications is required for accurately evaluating the performance of each alternative. Figure 4.4 shows the typical procedure of DES based supply chain optimization.

Developing the discrete event simulation model requires extensive gathering of input data,
model building, validation and verification. These simulation steps were time consuming and required simulation expertise, especially when modeling complex systems like SCNs.

4.5.3 Agent based modeling

There are several challenges in effective supply chain decision-making. The first challenge is that the information across all the departments and enterprises is distributed, dynamic, and disparate in nature. Secondly, in a present-day enterprise, decision centers reside in different departments. Thus, in the last few years, multi-agent systems have been a preferred tool for solving supply chain problems. Agents represent supply chain entities, e.g. customers,
manufacturers, and transportation. These agents use different interaction protocols and help in simulation of material, information, and cash flows. These interaction protocols are in the form of messages of various classes. Message handlers are associated with each message class and consider the agent receiving the message, to decide upon the message-processing semantics. The agents use various control policies to manage inventory, procure components, and determine optimal transportation routes.

In recent years, many researchers are also using an agent-based distributed modeling approach to model SCN [75, 76, 77, 78, 79, 80, 81]. One or several agents can be used to represent each entity in the SCN. Each agent is assigned with both a local objective and global objective. With the advent of mobile agents which can run on lighter platforms, the use of agents to both collect information and take decisions has become popular. Moreover, since maximizing own profit rather than the profit of the supply chain is preferred by companies, for network organizations the agent paradigm is a natural metaphor [75]. The multi-agent based modeling approach offers a way to elaborate the supply chain as the agents are autonomous and the agent rules can be defined in advance. The distributed decision nature of the multi-agent systems makes it easier to add other entities in the local environment. Entities leaving the SCN in the middle will not affect the entire SCN to a great extent. The agent rule framework provides a certain amount of trust among the partners as it eliminates the mistrust and deception among entities.

### 4.5.4 Mathematical programming based modeling

As shown in table 4.1, the mathematical programming based modeling approaches are used more at tactical and operational level. These quantitative approaches are used to increase the performance efficiency of the SCN. These mathematical approaches are mainly linear programming based methods and Mixed-integer programming based methods.

Linear programming can be used to model various situations, and identifies optimal problem solutions using linear mathematical equations. Only the relationships between decision variables and impact on objective functions are considered. Therefore, only problems expressed mathematically can be solved as there are only quantitative aspects but no qualitative ones. Computer support available with this technique is useful to solve more complex problems, like in case of many situations, where a wide range of constraints can be modeled. Although linear programming helps to find optimum solutions, it may not be realistic because of the dynamic and non-linear behavior of many variables.
Mixed-integer Integer programming is similar to the linear programming, but all the variables must be integers. In this approach, solutions are developing by still using the linear mathematical equations. On the other hand, Mixed-integer programming (MIP) can use a mixture of integer and real variables, to cover a wider variety of supply-chain modeling scenarios. Typically, integer or binary types are used for model configuration variables while the real variables relate to materials flow.

Arntzen et al [82] describes a mixed-integer programming model, called Global Supply Chain Model (GSCM) that incorporates a global, multi-product bill of materials for supply chains with arbitrary echelon structure and a comprehensive model of integrated global manufacturing and distribution decisions. Melachrinoudis and Min [83] used a dynamic, multiple objective, mixed-integer programming model for assessing the viability of a proposed facility site from multi-echelon supply chain perspectives and determining the optimal timing of relocation and phase-out in multiple planning horizons. Models of the supply chain under uncertainty generate large mixed-integer programming problems, which can make searching for solutions based on the standard MIP solution algorithms very time-consuming [83].

4.5.5 Petrinets based modeling

Petri nets allow one to model many structural and behavioral features typical for SCN. The concurrency and asynchronicity of process execution, hierarchical structure of SCN, different material and data flows, buffer sizes, transport assembly, production routes and machining operation times, and so on are possibly encompassed using Petri net models. Since these models are graphical and mathematical techniques to describe systems concurrency and synchronization, structural interactions and capturing precedence relations, they are regarded to be of unique interest and choice for SC modeling among the available discrete event formalisms [84]. In the context of operational models for SCN based on Petri nets, Viswanathan [85] employ generalized stochastic Petri nets to describe a particular example of SC and determine the decoupling point location, i.e., the facility from which all finished goods are assembled after customer order confirmation. More recently, Dotoli and Fanti [86] propose a generalized stochastic Petri nets model, describing a generic SC at an operational level in a modular and simple way, which is applied to a case study and tested by two management strategies. Moreover, in [87] a two product SCN is modeled by complex-valued token Petri nets and the performance measures are determined by simulation. These models are resource oriented, i.e., places are resources, tokens are jobs, namely parts/products and orders.
4.6 Comparison of the modeling approaches

There are a number of supply chain modeling methods that have been proposed and developed over the years depending upon the requirements. Each modeling approach has its own advantage and disadvantages. The selection of the modeling method also depends on the requirements of the application. In this thesis, the main focus is in identifying the suitable modeling approach to detect the nonlinear dynamics presented in the SCN network when subjected to uncertainties. The effects of the uncertainties are propagated in both upstream and downstream. The other key objective of this thesis is also mitigating the effect caused by the uncertainties on the SCN. In this context, the modeling approaches listed in section 4.5 are analyzed for their suitability.

4.6.1 Definition and justification of comparison criteria

The globalization of the operations of the SCN demands many new aspects in the modeling. The traditional modeling methods that are derived from different disciplines are partly successful in including some of these aspects. The primary aspect with the globalization is the uncertainties generated by the market fluctuations. We have seen in chapter 3, these uncertainties have long standing effects on the management of SCN and the product success in the market. This makes the primary comparison criteria as the ability of the modeling approach to include the industry and market induced uncertainties into the model. The market induced uncertainties propagate both upstream and downstream the SCN. This illustrates the importance of quantifying the effect of uncertainties or detecting the qualitative changes uncertainties brings into the model. As this is the main focus of this thesis, ability to quantify the dynamics or ability to detect the qualitative changes of the system is also taken as one of the comparison criteria. Detecting the effect of the uncertainties alone is not sufficient for the better management. The model should also able to include the control structure to control the effect of the uncertainties. The traditional modeling approaches described in section 4.5 are compared/analysed over these three important criterion.
4.6. COMPARISON OF THE MODELING APPROACHES

4.6.2 Ability to deal with uncertainties

Effective supply chains are those designed to deliver products and services promptly and reliably, at low cost and with high quality. Fluctuations in demand and production change dynamically, which makes supply chains complicated to understand. Many analytical models only take a few variables into consideration. The mathematical programming based modeling approaches especially depend on the clear definition of variables. To some extent discrete event simulation modeling approach considers the dynamic nature of variables. However, to consider the all the possible scenarios, discrete event simulation based modeling approaches demand high computational power and time. The system dynamics based approach is also similar to mathematical programming based approaches. The only feasibility the system dynamics based approaches give is the global dynamics of the SCN.

4.6.3 Ability to quantifying the effects of uncertainty

System dynamics based approach is very good to capture the global dynamics of the SCN. Petri nets based modeling also to some extent allow to capture the global dynamics by encompassing the hierarchical structure of SCN. The mathematical programming based approaches are highly useful for quantifying the dynamics in the specific problem setting but getting the global dynamics is a complex issue. However, to quantify the effects of uncertainties these modeling approaches are not suitable as the models themselves do not include the uncertainty aspect. These modeling approaches especially fall short when detecting the qualitative changes in the SCN induced by the uncertainties.

4.6.4 Scalability and adaptability of the models

Prior to the practical application of results, consideration of numerous constraints and additional issues is the main problem with most analytical models. Many analytical models are highly simplified, and consider only a few variables, such as inventory and the cost of running out of stock, ignoring other costs such as order processing and transportation. It means that, though gaining an understanding of general supply chain principles and effects using mathematical approaches may be valuable, too many simplifications are required by them to model realistic supply chain problems.
4.6.5 Inclusion of control structures

Traditional modeling approaches lack some modeling capabilities that are required for successful SCN simulation. Especially, the explicit modeling of control structures, i.e. the managers or systems responsible for control, their activities to control, may be considered as an intrinsic weakness of many simulation models and tools. Majority of the methods focus on transactions, leaving the definition of control structures largely to the analyst. As a result control structures are often hidden, as the analyst’s, controller, choice of building blocks does not appeal to SCN partners. It is clear that, nullifying the effects of the uncertainties in particular and the quality of decision making may be harmed severely, if the control structures that are intrinsic to SCM are left out.

The traditional modeling approaches detailed in this chapter are compared against these comparison criteria in table 4.2. System theory based modeling approach is well suited to deal with the uncertainties, but it lacks the ability to detect the qualitative changes caused by the uncertainties. System theory based approach also has the limitation to include the control structure. By considering this, in this thesis we propose to use the concepts from the dynamic systems theory in modeling and simulation of the SCN.

<table>
<thead>
<tr>
<th>Method</th>
<th>Ability to deal with uncertainties</th>
<th>Ability to quantifying the effects of uncertainty</th>
<th>Scalability and adaptability of the models</th>
<th>Inclusion of control structures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systems theory based modeling</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Discrete event modeling</td>
<td>Partial</td>
<td>Partial</td>
<td>No</td>
<td>Partial</td>
</tr>
<tr>
<td>Agent based modeling</td>
<td>Partial</td>
<td>Partial</td>
<td>Yes</td>
<td>Partial</td>
</tr>
<tr>
<td>Mathematical programming based modeling</td>
<td>Yes</td>
<td>Partial</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Petrinets based modeling</td>
<td>Partial</td>
<td>Partial</td>
<td>No</td>
<td>Partial</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of traditional modeling approaches

4.7 Summary

In this chapter, the objectives of modeling the supply chain are detailed. The major requirements for modeling the SCN are outlined and their detailed description is carried out based on
4.7. SUMMARY
	heir importance. State-of-the-art modeling methods in SCN are detailed by classifying them into modeling frameworks by standard organizations and simulation based modeling approaches that are developed to analyze and optimize supply chains. Different modeling approaches are compared over a set of comparison criteria, such as ability to quantifying the dynamics, ability to deal with uncertainties, scalability and adaptability of the models, to list their merits and demerits and to show the necessities for nonlinear dynamic systems based modeling and simulation of SCN.
Chapter 5

Basics of dynamic systems

5.1 Dynamic systems definition

The theory of dynamic systems provides a paradigm for modeling and studying the phenomena that undergo spatial and temporal evolution. Dynamic system is the mathematical formalization of the dynamics of the time dependence of a point’s position in its state space. In other words it can be explained as the rule for time evolution on a state space \[88\]. A dynamic system consists of mainly three major components namely \[26\]:

- A setting in which the dynamical behavior takes place.
- Mathematical rules which specifies the evolution of the system in time.
- Initial condition or the initial state from which the system starts.

The concepts from the dynamic system theory provide robust methods/tools to analyze the behavior of complex systems over time and space. The methods/tools provide the qualitative solutions for the properties of the dynamic systems such as \[26\]:

- The system solutions dependence on the initial conditions.
- The system solutions dependence on the system parameters.
- The system solutions dependence on the mathematical formulation of the dynamics of the systems.

5.1.1 Linear dynamic systems

Linear dynamic systems make a continuous changes in the behavior without any change in the sequence \[88\]. A continuous time dynamic system is called a linear system when it can be written
in the form given in Eq. 5.1

\[ x(t) = Ax(t) + c, \quad x, c \in \mathbb{R}^n, \quad t \in \mathbb{R} \] (5.1)

// (t) is the n dimensional vector of state variables at the point in time t
\[ x(t) = \frac{dx(t)}{dt} \] is the vector of time derivatives of the state variables
A is an nxn matrix of constant coefficients
\[ c \] is an n dimensional column vector of constants

A discrete time dynamic system is called a linear system when it can be written in the form given in Eq. 5.2

\[ x_{t+1} = Ax_t + c, \quad x, c \in \mathbb{R}^n, \quad t \in \mathbb{Z} \] (5.2)

\[ x_t \] is the vector of state variables in the discrete period
A is an nxn matrix of constant coefficients
\[ c \] is an n dimensional column vector of constants

### 5.1.2 Nonlinear dynamic systems

Unlike linear dynamic systems, the nonlinear dynamic system does not satisfy the superposition principle, i.e. output is not proportional to its input [89]. A continuous time dynamic system is called a nonlinear system when it can be written as n dimensional ordinary differential equation system.

Consider the n dimensional ordinary differential equation system defining the motion of the state variables \( x_i, i = 1, ..., n^3 \).

\[ \dot{x}_1 = f_1(x_1, ..., x_n), \]
\[ \vdots \]
\[ \dot{x}_n = f_n(x_1, ..., x_n), \] (5.3)

or, in vector notation,

\[ \dot{x} = f(x), \quad x \in \mathbb{W} \subset \mathbb{R}^n, \] (5.4)

\[ \mathbb{W} \] as an open subset of \( \mathbb{R}^n \)
A dot over a variable denoting the operator \( d/dt \)
The functions \( f_i, i = q, ...n \) are usually assumed to be \( C^\infty \)
5.2. KEY CONCEPTS TO UNDERSTAND THE DYNAMIC SYSTEMS BEHAVIOR

Differential system like (5.3) describe the vector fields in \( W \), i.e. for each \( x \in W \) the dynamic system unambiguously determines the direction and the speed of change of that point.

In a nonlinear system the components are interactive, interdependent, and exhibit feedback effects [89]. A nonlinear dynamical system consists of a set of possible states such that the knowledge of these states at some time \( t = t_0 \), together with the knowledge of an external input to the system \( t < t_0 \), completely determines the behavior of the system any time \( t > t_0 \).

5.2 Key concepts to understand the dynamic systems behavior

Nonlinear dynamic systems exhibits various dynamic states. The dynamic system theory has involved qualitative solutions for the properties of the dynamic systems. In order to better understand the meaning of the qualitative solution, one has to clearly understand the basic definition of key concepts of dynamic systems. In this section, we introduce the key concepts that are required to understand the behavior of dynamic systems.

5.2.1 Phase space

The qualitative analysis of differential equations was first developed by Henri Poincare with further developments given by Birhoff. Henri Poincare introduced the phase space analysis, a graphical method for studying second order systems. The basic idea of the method is to generate, in the state space of a second order dynamic system (a two-dimensional plane called the phase plane), motion trajectories corresponding to various initial conditions, and then to examine the qualitative features of the trajectories. In such a way, information concerning stability and other motion patterns of the system can be obtained. The ”phase portrait/diagram” is the set of all possible states of a dynamical system; each state of the system corresponds to a unique point in the phase space [90]. Considering two system variables, \( X \) and \( Y \), we can plot one variable against the other at a given point in time on a standard XY graph. This is called a ”phase portrait/diagram” of the system. The form and structure of the phase portrait reveals information about the system behavior for the chosen parameter values: in case of the form of a point, the system is in saturation; in case of a periodic orbit, the system is in regular state; and in case of discernible patterns, the system is in a chaotic state [90].

Phase space analysis has a number of useful properties as shown below:

- As a graphical method, it allows us to visualize what goes on in a nonlinear system starting
from various initial conditions, without having to solve the nonlinear equations analytically.

- It is not restricted to small or smooth nonlinearities, but applies equally well to strong nonlinearities and to "hard" nonlinearities.

- Practical control systems can indeed be adequately approximated as second-order systems, and the phase plane method can be used easily for their analysis.

The fundamental disadvantage of the method is that it is restricted to second order (or first order) systems, because the graphical study of higher-order systems is computationally and geometrically complex [88].

Figure 5.1 shows the movement of the pendulum in real space, marking its position left or right with its corresponding height. The movement of the pendulum along the position dimension and the velocity dimension can’t be visualized in the real space. However as shown in figure 5.2 the location of the pendulum can be marked in the phase space along both the position dimension and velocity dimension. The importance of the phase space diagram is tracking the evolution of the dynamic system over time as shown figure 5.2.

5.2.2 Attractor

The important notion in the description of nonlinear dynamic systems is the notion of an attractor [89, 26]. An attractor is defined as a set of states to which the dynamical system tends to evolve over a long enough time, regardless of the starting conditions. An attractor is an example of an invariant set, set that evolves to itself under the dynamics, with specific
5.2. KEY CONCEPTS TO UNDERSTAND THE DYNAMIC SYSTEMS BEHAVIOR

Figure 5.2: The movement of pendulum in phase in comparison with real space

properties [26]. The attractor is in different forms. The details of these forms will be discussed in next section.

5.2.3 Types of attractors

There may be several attractors in a model. Each attractor has a domain of attraction. The trajectory of the model converges to that attractor in which domain initial conditions were located [89, 26].

Fixed point attractor

For a long time, economics has concentrated on a special kind of attractor, namely fixed point attractor. Fixed point attractor are also called equilibrium point attractors and exhibit the saturation state [89]. The saturation manifests itself by a sudden exhibition of fixed or constant values/data along each level of the SCN. When the state of saturation is obtained, further changes/flows in the supply chain are not represented effectively. Zero net force results in saturation state as forces acting on an object cancel out each other. In the case of the system of sink, the drain represent the fixed point attractor. The fixed point attractor can be depicted as shown in figure 5.3.
Limit cycle

A limit cycle is a periodic orbit of the system that is isolated [89, 26]. In phase space diagram, it represents as a closed trajectory with at least one other trajectory merging into it after the long enough time as shown in figure 5.4. If all the neighboring trajectories merge into this limit cycle as the time approach infinity or long enough, then it is called stable limit cycle.

Limit tori

Dynamic systems also have the possibility of multiple closed orbits which are alternatively stable and unstable. However, the question of how many cycles exist in a dynamical system is important, because in case of multiple cycles the initial conditions determine the final motion of the system with specific amplitude. It is important to know, especially in economic and supply chain related models, whether by choice of the initial conditions the amplitude of the cyclic motion can be decreased or not. In some dynamic systems, there may be more than one
5.2. KEY CONCEPTS TO UNDERSTAND THE DYNAMIC SYSTEMS BEHAVIOR

frequency in the trajectory of the system through the state. If two of these frequencies form an irrational fraction (i.e. they are incommensurate), the trajectory is no longer closed, and the limit cycle becomes a limit torus [26](see figure 5.5).

![Figure 5.5: A limit tori attractor](image)

**Strange (chaotic) attractor**

Dynamic systems cannot be attributed to the possibility of displaying the cyclic periodic patterns. They can also exhibit the behavior of variables that strongly resembles random process. The other important attractor in the study of the dynamic systems is the strange attractor. The attractor is called strange as the trajectories in the phase space do not follow any patterns [89] as shown in figure 5.6. It is not possible to predict/infer the future development of

![Figure 5.6: A strange attractor](image)

system with precision. This unexpected property of nonlinear system with discernible patterns in phase space is called as the chaotic behavior. The numerical investigation of the dynamical systems uncovered the unexpected behavior in the dynamic systems. Lorenz when conducting
the experiments with fluid convection model first time identified this strange attractor for a small change in initial conditions. It is obvious that random like behavior in nonlinear is at least theoretically relevant to business systems. Especially in management of SCN the external random influences are therefore assumed to being superimposed on the regular motion of flows in the SCN.

5.2.4 Basin of Attraction

![Figure 5.7: Basin of attraction of attracting set A in case of single point attractor](image1)

An attracting set is a set to which trajectories starting at initial points in a neighborhood of the set will eventually converge. The set of all initial points which are attracted by $A$ is called the basin of attraction of $A$. The basin of attraction is the sink’s basin when the attractor can be considered as the drain at the bottom with the imagination of a complex system as a sink. The shaded areas in figures 5.7 and 5.8 depict basins of attraction for the cases in which the attractor is a single point (figure 5.7) and in which the attractor is a closed curve (figure 5.8). The basin of attraction is delimitated by the basin boundary [89]. In most parts of this report the term ”attracting set” will be identified with the term attractor.

![Figure 5.8: Basin of attraction of attracting set A in case of closed curve attractor](image2)
5.3 Different possible states of a dynamic system

We have seen in previous section the existence of different type of attractors in the dynamic systems. This also means that dynamic systems also undergo different states through their evolution. Dynamic systems display different state behaviors as the simulation time approaches to infinity or long enough. In this section we detail the states of the dynamic system that are important to understand this report.

5.3.1 Transient state

![Figure 5.9: Steady state of the pendulum](image)

Transient as per the Oxford dictionary means staying or working in a place for a short time only. Any system that is exhibiting the transient behavior is called as that the system is in transient state [91]. For example the pendulum system when disturbed is in transient state some time before again attaining its regular oscillations as shown in figure 5.9. Figure 5.10 shows the motion of the pendulum in transient state in real space.

![Figure 5.10: Transient state of the pendulum](image)
5.3.2 Steady state

Dynamic system is said to be in steady state if the behavior of the system stay same for the long enough time. The system will come to steady state after passing the transient state. For example, in the case of the pendulum, the system is said to be in steady state if the pendulum is oscillating regularly with same frequency or rest [89]. The steady state behavior is the important behavior in studying the dynamic systems. The change of the steady state behavior due to the change in initial conditions or change in the external conditions is an important aspect in analyzing the nonlinear dynamic systems.

Regular state

The trajectory of dynamic system in steady state takes different form depending upon the attractor. The steady state behavior of the system describes whether the system is stable or not stable. The pattern of the attractor defines the state of the dynamic system. In most economic and business applications, especially when dealing with nonlinear dynamical nature of the systems, the pattern of trajectory is important. If the trajectory of the system takes cyclic pattern after the transient state, the system is said to be in regular state [89, 26]. The regular state of the system facilitates the future prediction/inference of the system based on the phase space portrait. The phase space portrait of the regular state of the pendulum is the circle as shown in figure 5.2.

Saturation state

The saturation state or equilibrium is a state in which opposing forces or influences are balanced or canceled out. In some of the physical systems it is desirable state but in economical and business systems it is not a desirable state. The equilibrium state of the pendulum is represented with fixed point in the phase space as shown in figure 5.11. This is the position of the pendulum when it is at rest or calm. The saturation state is characterized with constant or fixed data [89, 26]. This nature makes it as one of the dangerous states of the system in SCN management.

Chaotic state

Chaotic state as the name suggests refers to the state of lacking order or predictability. Chaotic systems look like random systems but in contrary they are deterministic in nature governed by mathematical rules. Mathematically, chaos means deterministic behavior which is very sensitive
5.4. INTRODUCTION TO BIFURCATION THEORY

Figure 5.11: Saturation state of the pendulum

to its initial conditions [9]. In other words, the small change in the initial conditions can lead the trajectory differ largely as shown in figure 5.12. Chaos is defined as an aperiodic, unpredictable and bounded dynamic in a deterministic system with sensitivity dependence on initial conditions (Crawford, 1991). Chaos is a disorderly long-term evolution occurring in deterministic non-linear systems. Chaotic state is defined precisely by Edward N. Lorenz [9] based on their trajectories in the phase space as follows:

- No matter how close two distinct aperiodic trajectories come to each other, they must eventually move away from each other.
- Every possible aperiodic trajectory moves arbitrarily close to every other one.
- If an aperiodic cycle approximates a cycle of order $k$ for a while, it must move away from that cycle.

Figure 5.12: Chaotic state of the pendulum [9]

5.4 Introduction to Bifurcation theory

The word bifurcate means divide into two branches or forks. The word bifurcation is widely used to describe any situation in which the qualitative, topological picture of the object we
are studying alterations with a change of the parameters on which the object depends. Bifurcation theory is a subject with classical mathematical origins. Computer-assisted studies of dynamics and infusion of new ideas and methods from dynamical systems theory resulted in tremendous change in Bifurcation theory in the recent past [65]. The bifurcation phenomena can be related to the notion of structural stability. In other words, a dynamical system is called structurally stable if the qualitative properties of the dynamic system persist with small variations in its structure. As we have seen in section 5.2, the phase portrait provides a global qualitative picture of the dynamics, and this portrait depends on any parameters that enter the equations of motion or boundary conditions. If one varies these parameters, the phase portrait may deform slightly without altering its topological features, pattern of trajectory, or sometimes the dynamics significantly change the topological changes in the phase portrait. The bifurcation is a qualitative change observed in the behaviour/state of a system as its parameter settings vary. The bifurcation is observed if the state of the system suddenly changes qualitatively upon small/smooth variation of the parameter values. Bifurcation theory studies these qualitative changes in the phase portrait (for example, regular state, saturation state and chaotic state) [65].

5.4.1 Bifurcation analysis

The methods and analysis of the results of the bifurcation theory are fundamental to the understanding of the nonlinear dynamical system. The bifurcation analysis aids in understanding the bifurcation scenarios with the aim of defining/determining the states (equilibrium/fixed points, periodic or chaotic states) of the system in a given parameter space. Bifurcation analysis is very significant in the analysis of business system, especially SCNs’, as it is usually impossible to assign a definitive, once-and-for-all valid number to most parameters occurring in the complex dynamical system such as SCN. It is highly desirable to determine whether the qualitative behavior of a dynamical system persists under the variations in the parameter space. Bifurcation diagrams are used to show the possible evolution of the system in long term (for example, regular state, saturation state and chaotic state) as a function of the parameter of the system [65].

5.4.2 Interpretation of bifurcation diagrams

The bifurcation diagrams allows a comparison between different states of the system in a given parameter space. In order to better understand the interpretation of the bifurcation diagrams we consider the an example of the bifurcation diagram of the logistic map as shown in figure
5.13. A logistic map is mathematically defined as shown in Eq. 5.5.

\[ x_{n+1} = rx_n(1 - x_n) \]  \hspace{1cm} (5.5)

The state of \( x \) in the logistic map depends on the parameter \( r \). The Eq. 5.5 is a very good example to show how the complex and chaotic behaviors can be generated from a simple nonlinear equation. The dependence of the behavior of the state of \( x \) on the parameter space of \( r \) is shown in figure 5.13. As shown in figure 5.13, the state of \( x \) is stable on the in the parameter space of \( r \) between 2 and 3. The state of \( x \) is also stable for \( r \) between 3 and 3.45. The only change is that the \( x \) may oscillate between two values. For \( r \) between 3.45 and 3.54 also the system is stable and \( x \) may oscillate between 4 values forever. But beyond \( r \) equal to 3.54 the system exhibits the chaotic state by displaying a different value for each run. However, in the figure 5.13 one can observe the system is stable at some points event after 3.54 by oscillating between 8, 16, 32 values. These stable points are also called as islands of stability.

5.5 Types of synchronization in nonlinear systems

Synchronization is common word encountered in day-to-day language and scientific language. Originating from the Greek words in a direct translation mean "sharing the common time" or "occurring in the same time" [92]. Although there have been several attempts to define the synchronization in the context of dynamic systems, no successful definition of synchronization appears [93] in literature. Kurts defined [93] synchronization as an adjustment of rhythms of oscillating objects due to their weak interaction. However, synchronization is defined by many other depending upon the type of synchronization. Over the last decade, a number of
new types of synchronization such as complete synchronization, phase synchronization, lag synchronization and general synchronization have been appeared.

5.5.1 Complete synchronization

This is the most frequently discussed form of synchronization within the nonlinear dynamic systems. Here in this the two sub-systems, $X(t)$ and $Y(t)$, are identical both in amplitude and phase as shown in figure 5.14. The properties of phase space variables and more clearly the phase space structure of the trajectory are identical for the two sub-systems. Most discussions in the literature use the Eq. 5.6 as a comparison function between the two sub-systems.

$$\lim_{t \to \infty} |X(t) - Y(t)| = 0$$  (5.6)

![Figure 5.14: Complete synchronization of two sub-systems](image)

5.5.2 Generalized synchronization

![Figure 5.15: Generalized synchronization of two sub-systems](image)
5.5. TYPES OF SYNCHRONIZATION IN NONLINEAR SYSTEMS

In the generalized synchronization there exists a functional relationship between the amplitudes and the phases of the sub-systems, \( X(t) \) and \( Y(t) \), as shown in Eq. 5.7.

\[
y(t) = F(x(t))
\]  

(5.7)

By constructing appropriately nonlinear coupling terms, some sufficient conditions for determining the generalized synchronization between the two subsystems will be derived. The generalized synchronization of two sub systems in real space is as shown in figure 5.15.

5.5.3 Phase synchronization

Phase synchronization involves the sub-systems \( X(t) \) and \( Y(t) \), properties called phases. The amplitude of the sub-systems may be different as shown in figure 5.16. If the measured properties are given by \( g(x) = \phi_x(t) \) and \( g(y) = \phi_y(t) \) then the most common comparison function is,

\[
h[g(x), g(y)] = U[\epsilon, (g(x) - g(y))] 
\]  

(5.8)

Here \( U(u, v) \) is a vector with \( \alpha \)-th component \( U_\alpha(u, v) = \Theta[u_\alpha - |v_\alpha|] \), and \( \Theta \) is the unit step function. Eq 5.8 says that synchronization means \( |\phi_x\alpha - \phi_y\alpha| < \epsilon_\alpha \), so \( \epsilon \) is the maximum tolerable separation between the components of the phase.

5.5.4 Lag synchronization

Lag synchronization is also similar to the phase synchronization. In addition to the phase synchronization, the amplitudes of the sub systems are also identical as shown in the figure 5.17. The Eq. 5.9 is used as a comparison criteria for the lag synchronization.

\[
\lim_{t \to \infty} |X(t) - Y(t \pm \tau)| = 0
\]  

(5.9)
5.6 Summary

The necessary basics to understand the nonlinear dynamic systems are presented in chapter 5. The linear and nonlinear dynamic systems are introduced for the better understanding of the supply chain model in next chapters. The important concepts of dynamic systems such as phase space, attractors, basin of attraction are defined and described in detail in this chapter. The phase space analysis is used for the qualitative analysis of the second order systems. Various possible states of a dynamic system such as regular state, saturation state and chaotic state are explained in detail. The description of the definitions of various states and the key concepts of the dynamic systems help in understanding the possible nonlinear dynamic states of the SCN. The other important concept from the dynamic systems used in this thesis is bifurcation theory. In this chapter, the basics of bifurcation theory are introduced. The interpretation of the bifurcation diagrams is presented with the illustration of the logistic curve bifurcation diagram.
Chapter 6

Nonlinear dynamic systems based modeling of supply chains

6.1 Nonlinear dynamics in supply chains

SCNs with its complex nature exhibits various dynamic states. The simple supply chain network illustrated through beer distribution game at MIT exhibited various dynamic states under the influence of uncertainties as we have seen in chapter 3. The beer game simulations only showed how and why supply chains exhibit various non-stable states under the influence of internal uncertainties and fluctuations caused by uncertainties even when players are with experience and highly educated. The modern supply chains under the influence of uncertainties, both indigenous and exogenous, exhibit various dynamic states such as saturation and chaos. Wilding [22] showed that the SCNs never remain in a regular state. The small fluctuations, perturbations, generated due to the uncertainty always prevent SCNs’ in achieving the regular state.

6.2 Coupled oscillatory systems based modeling of the supply chain

Simulation of the SCN systems and the analysis of the nonlinear dynamics presented in the system at various parts is essential in designing the SCN, in re-engineering the SCN and in the operational efficiency of the SCN. If the model generates undesirable states, the real system with more complexities has very high chance to exhibit the undesirable states. In recent years, there has been considerable interest in modeling the supply chains using nonlinear models [22, 57, 67, 94, 69]. The growth in computational power and the continuous development of
CHAPTER 6. NONLINEAR DYNAMIC SYSTEMS BASED MODELING OF SUPPLY CHAINS

Theoretical models of the supply chain dynamics has resulted in an increasing dissatisfaction with the approximations that are provided by linear systems. Many of the models described in the literature are particularly interested in capturing the nonlinear features of the data. The account of nonlinearity of the links between different players in the supply chain and the interaction of the players within the SCN leads to very complicated mathematical problems. This coupled nature between the players within the SCN makes the coupled oscillatory systems based modeling a realistic model for modeling the SCN. By considering these issues, we envisage the complex non-linear modeling of a three-echelon supply chain to represent the realistic dynamics using coupled oscillatory systems approach.

6.2.1 Description of the supply chain model

The theoretical framework for SCM underlies the setting, optimization and control of the system model. The system model is not unique for all the supply chains [69]. The system dynamics change for each type of product, for example, food, oil, consumer goods, etc., depending upon the processes involved. The system dynamics based approach to model the business dynamics was first introduced by Forrester [7]. System dynamics understands the dynamic behavior of complex physical and social systems using an approach based on perspection of delays and information feedback as it has its origins in management and control engineering. System dynamics is an approach which is actively used to model the managerial behavior.

A three level model is envisaged, as shown in figure 6.1 to describe a simple scenario in a very complex supply chain. The model consists of the manufacturer, distributor and retailer, the three key players in the SCN. The dynamics between the suppliers and manufacturers are incorporated into the manufacturer. The model also resembles the MIT beer distribution game model in some aspects. The manufacturer holds the safety stock in order to reduce the small production batches and to meet the growth in demand. The nonlinear supply chain models in

![Theoretical three level model of the SCN](image)
the literature [94, 69] focus mainly on specific tasks, and thus become a transaction-oriented approach. In this work, we mainly focus on building a non-linear supply chain model that can exhibit more complexity at different stages in the supply chain in addition to the inventory concept at all stages.

In this work, we particularly include the information distortion rate at the distributor as it is one factor that is influenced by uncertainties from the retailer. The demand information is distorted before presenting it to the manufacturer as a means for the avoidance of the uncertainties generated by retailer. The retailer also uses his intuition and experience to avoid the uncertainties that come from the customer and distributor. The customer order satisfaction rate is used as a means to avoid the uncertainties generated by the customer. The rate of order satisfaction of customer is also used to mitigate the uncertainties from the distributor. Retailer uses this coefficient as a measure to sustain in the market. Manufacturer uses the safety stock as a means to avoid uncertainties from both upstream and downstream of the SCN. As this thesis mainly focus on mitigating the effects of uncertainties, these three key parameters at the three key players are incorporated into the model to bring reality to the model.

The notations introduced to facilitate the description of the model are shown in table 6.1.

<table>
<thead>
<tr>
<th>i</th>
<th>Time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>Rate of customer demand satisfaction at retailer</td>
</tr>
<tr>
<td>r</td>
<td>Rate of information distortion of products demanded by retailer</td>
</tr>
<tr>
<td>k</td>
<td>Safety stock coefficient at manufacturer</td>
</tr>
<tr>
<td>x_i</td>
<td>The quantity demanded by retailer in current period</td>
</tr>
<tr>
<td>y_i</td>
<td>The quantity distributors can supply in current period</td>
</tr>
<tr>
<td>z_i</td>
<td>The quantity produced in current period depending on the order</td>
</tr>
</tbody>
</table>

Table 6.1: Variables used in the theoretical model of the SCN

The model describes the dynamics involved in the SCN. The model also takes into consideration that the players individually make decisions that benefit them.

6.2.2 Requirements and assumptions of the supply chain model

The key assumption in modeling the system described in figure 6.1 is that the demand information, i.e. information between retailer, distributor and manufacturer, is transmitted along the links of the supply chain in a unit time. The information distributors or producers deal within period \( i + 1 \) is what retailers or distributors requested in period \( i \). This delay in information processing leads to the information distortion.
The other important requirement is considering the fulfillment of market demand. The quantity demanded for the products in the current period always depends on the demand satisfaction in the previous period. This is a key requirement because this shows the ordering pattern upstream. The players in the SCN make their decisions on the orders and their inventories in current period after considering information distortion and their present inventory levels.

### 6.2.3 Mathematical formulation of the supply chain model

The mathematic formulation of the dynamics shown in figure 6.1 is explained in detail in this section. We consider that the demand information is transmitted within the layers of the supply chain with a delay of one unit of time. As illustrated in figure 6.1, the ordering quantity is not the same as the requested order quantity at any level. The order quantity at the current period of time at the retailer is linearly coupled with the distributor and it is influenced by how much demand is satisfied in the previous period of time. This scenario/phenomenon is modeled by Eq. 6.1:

\[ x_i = m(y_{i-1} - x_{i-1}) \]  \hspace{1cm} (6.1)

Here \( m \) is the ratio at which the customer demand is satisfied.

The dependency/coupling between the distributor, the producer and the retailer (figure 6.1) is not linear. Indeed the distributor needs to take the combined effect of the retailer and producer into consideration before making his order, i.e., quadratic coupling. Apart from this, the distributor also needs to take into consideration that the order information received from the retailer might be distorted. This scenario is modeled by Eq. 6.2:

\[ y_i = x_{i-1}(r - z_{i-1}) \]  \hspace{1cm} (6.2)

Here, \( r \) is the information distortion coefficient.

The production quantity from the producer unit typically depends on the distributor’s orders and the safety stock. However the distributor’s orders again depend on the retailer’s orders, i.e., the producer needs to take the combined effect of retailer and distributor into account before making production decisions. This scenario is modeled by Eq. 6.3:

\[ z_i = x_{i-1}y_{i-1} + kz_{i-1} \]  \hspace{1cm} (6.3)
6.2. COUPLED OSCILLATORY SYSTEMS BASED MODELING OF THE SUPPLY CHAIN

Eq. 6.1 - 6.3 represent the three coupled systems each representing:

- The quantity demanded by customers (Eq. 6.1).
- The inventory level of distributors (Eq. 6.2).
- Quantity produced by producers (Eq. 6.3)

where:

- $x_i < 0$ denotes that the supply is less than customers demand in the previous period.
- $y_i < 0$ denotes that the information is severely distorted and no adjustment is necessary at the inventory level.
- $z_i < 0$ denotes the cases of overstock or return and hence no new productions.

Eq. 6.1 - 6.3 are discrete models describing the dynamics of the SCN of figure 6.1. Considering very small time intervals, the continuous model in Eq. 6.4 can be derived from Eq. 6.1- 6.3.

\[
\begin{align*}
\dot{x} &= my - (m + 1)x \\
\dot{y} &= rx - y - xz \\
\dot{z} &= xy + (k - 1)z
\end{align*}
\]  

(6.4)

If the conditions are satisfied, Eq. 6.4 leads to the Lorenz model in Eq. 6.5.

\[
\begin{align*}
\dot{x} &= \sigma(y - x) \\
\dot{y} &= rx - y - xz \\
\dot{z} &= xy - bz
\end{align*}
\]  

(6.5)

From the theory of dynamic systems it is proved that this model produces a wide variety of non-linear features depending upon the parameters’ values. This model is particularly of interest when dealing with the modeling of scenarios/phenomena which are very sensitive to initial conditions and to uncertainties as well. In the case of SCN uncertainties, when added at one layer effectively propagate in both upstream and downstream. These are the common dynamics exhibited by realistic SCNs. A similar model (Eq. 6.5) is also proposed by the authors [67] to exhibit the supply chain dynamics and mitigate the bullwhip effect.
CHAPTER 6. NONLINEAR DYNAMIC SYSTEMS BASED MODELING OF SUPPLY CHAINS

6.3 Nonlinear dynamics of the supply chain model

The theory of nonlinear dynamic systems shows that the systems change their states considerably with a small change in initial conditions. Lorenz, in 1960, through his seminal work showed that small differences in a dynamic system, a model of the atmosphere, could trigger vast and often unsuspected results such as storm [9]. SCNs with their coupling nature and hidden complex structures are also dynamic systems. The SCN under the influence of the external fluctuation, external perturbation, is also a representative case for the dynamic system as we have seen in chapter 1 with a simple beer distribution SCN. Before examining the dynamics of the SCN due to uncertainties caused by the external fluctuations, we describe an example SCN.

6.3.1 Description of the example supply chain from the supply chain model

Before considering the effects of external perturbations, let us focus first on discussing example supply chain with the given parameter values. The three key parameters we have incorporated into our model to deal with the uncertainties, safety stock coefficient at manufacturer, information distortion at distributor and customer demand satisfaction at retailer should be fixed to a realistic values in order to see the performance of the supply chain under normal operational conditions. The safety stock coefficient, $\sigma$, at the manufacturer is fixed to 15, i.e. the manufacturer produces 15% more than the order received in order to face the uncertain situations. The distributor assumes that the information received from the retailer is distorted, $r$, at around 30%. The distributor takes this fact into account before placing the order with manufacturer. The retailer places orders with an assumption that at least two third of his orders are satisfied by the distributor ($b = 2/3$). This in turn affects the customer satisfaction ratio for retailer. These three key parameters are fixed to study the dynamics of the system under the stable condition and we call this system example "reference supply chain model".

6.3.2 Steady state of the supply chain model

The phase portrait of the reference supply chain model described in previous section with $\sigma = 15$, $r = 30$ and $b = 2/3$, is studied to understand the behavior of the supply chain without the influence of external uncertainties. The form and structure of the phase portrait reveals information about the system behavior for the chosen parameter values: in case of the form of a point, the system is in saturation; in case of a periodic orbit, the system is in regular state; and in case of discernible patterns, the system is in a chaotic state [26]. Figure 6.2 displays the phase portrait of the reference supply chain system, whereby Ref, Ref, Ref represent the $x$, $y$ and $z$ of the three-level supply chain described in Eq. 6.5. The phase portrait reveals that
with the assumed parameters for the reference supply chain, the supply chain is in the regular state. The three key players involved in managing the SCN have their objectives met without affecting the other players.

Figure 6.2: Phase space representation of the reference supply chain with $\sigma = 15$, $r = 30$ and $b = 2/3$

### 6.3.3 Saturation state of the supply

Due to the dynamic changes as discussed (in time domain) in chapter 3, some pre-defined settings or requirements within the supply chains (thresholds such as safety stocks) may be varying accordingly as a consequence of these uncertainties. It should be worth mentioning that a combination of the simultaneous effects of both internal and external infertilities may be responsible for the dynamic motion variations (for example flow of materials and information exchange) within the supply chain. This is a concrete and/or realistic scenario as the supply chains of many companies are currently exposed to both types of uncertainties. However, the analysis in this work is restricted to the case where the reference model is subjected to external perturbations due to the external uncertainties.

We consider the external perturbations to be nonlinear as the market-place behavior is nonlinear in nature. Assuming that the perturbations can affect any of the three levels of the SCN, a perturbed form of Eq. 6.5 is proposed in Eq. 6.6.
\[
\begin{align*}
\dot{x} &= \sigma(y - x) + d_1 \\
\dot{y} &= r\dot{x} - \dot{y} - x\dot{z} + d_2 \\
\dot{z} &= \dot{x}\dot{y} - bx + d_3
\end{align*}
\] (6.6)

Where \(d_i (i = 1, 2, 3)\) represent the external perturbations.

Further investigations have been performed to analyze how the supply chain behavior changes for various external perturbations. Figure 6.2 shows the phase space structure of the reference supply chain subjected to the external perturbations with \(X(\text{ext, int}), Y(\text{ext, int}), Z(\text{ext, int})\) representing \(x, y\) and \(z\) of the supply chain described in Eq. 6.6. External perturbation \(d_1 = 10\cos(5t), d_2 = 5\cos(10t)\) and \(d_3 = 10\cos(10t)\), is particularly considered as non-linear perturbation as the external uncertainties (for example, market fluctuation and forecasting demand) exhibit nonlinear dynamics. This figure 6.3 shows that the supply chain subjected to external perturbations can exhibit the state of saturation which is characterized by non-dynamic (or fixed) data in each level of the three levels. The saturation manifests itself by a sudden exhibition of fixed or constant values/data along each level of the externally perturbed three echelon SCN. When the state of saturation is obtained, further changes/flows in the supply chain are not represented effectively.

### 6.3.4 Chaotic state of the supply chain

Further investigation into the dynamics of supply chain with different external perturbations is conducted to analyze the SCN behavior. Figure 6.4 shows the phase space structure of
6.4. BIFURCATION ANALYSIS OF THE SUPPLY CHAIN MODEL

the reference supply chain subjected to the external perturbations with $X_{\text{ext, int}}$, $Y_{\text{ext, int}}$, $Z_{\text{ext, int}}$ representing $x$, $y$, and $z$ of the supply chain described in Eq. 6.5. External perturbation $d_1 = 0.56\cos(3t)$, $d_2 = 20\cos(5t)$ and $d_3 = 50\cos(10t)$, is particularly considered as nonlinear perturbation as the external uncertainties (for example, market fluctuation and forecasting demand) exhibit nonlinear dynamics. The effects of external perturbations on the original (or specific) requirements of the reference supply chain are clearly shown by the well-known chaotic Lorenz attractors exhibited by the reference SCN subjected to perturbations. This structure in the phase space shows the occurrence of the well-known bullwhip and chaotic effects in the three-level supply chain by displaying discernible patterns.

![Figure 6.4: Chaotic state of the supply chain due to external perturbations](image)

6.4 Bifurcation analysis of the supply chain model

The bifurcation is a qualitative change observed in the behaviour/state of a system as its parameter settings vary. The bifurcation is observed if the state of the system suddenly changes qualitatively upon small /smooth variation of the parameter values. The bifurcation theory [65] is the analysis/study of the bifurcation scenarios with the aim of defining/determining the states (equilibrium/fixed points, periodic or chaotic states) of the system in a given parameter space. Basically, bifurcation values/points are critical values leading to qualitative changes in the states of the system.

In section 6.3, we have seen the dynamic states of the supply chain network subjected to the external perturbations. In order to mitigate the effects caused by the uncertainties, the
players in the supply chain respond by increasing the inventories and changing the coefficients of information distortion, customer satisfaction rates. However, this change in the parameters that are under their control discrepantly can effect the global supply chain performance. From the theory of dynamic systems, it is proven that the change in tiny/small variations of the parameters can again lead the system to exhibit various dynamic states. Bifurcation analysis can help to find the "islands of stability" for these key parameters which can be used to mitigate the effect of external uncertainties. Figures 6.5 - 6.6 are bifurcation diagrams showing the

![Figure 6.5: Bifurcation plot showing the sensitivity of the supply chain to the change of internal variable \( \sigma \) (\( d\sigma \))](image)

states of the perturbed supply chain. Figure 6.5 shows the sensitivity of customer satisfaction parameter over the all the three players (X(ext, int), Y(ext, int), Z(ext, int)). Similarly, figure 6.6 shows the rate of information distortion over the supply chain network. The analysis is performed and the states presented in the SCN are observed by changing the parameters from their regular state (\( \sigma = 15 \) and \( r = 30 \)) to the possible variation window range (\( 0 \leq d\sigma \geq 50 \) and \( 0 \leq dr = \geq 50 \)) of the supply chain network. Figures 6.5 - 6.6 show the extreme sensitivity of the supply chain to the variations of \( \sigma \) and \( r \). Indeed, windows of regular states are shown which alternate with windows of chaotic states (for example period-1, period-3, and chaotic attractors are shown). From Figures 6.5 - 6.6, windows of parameters can be defined in which each of
these states can occur.

Figure 6.6: Bifurcation plot showing the sensitivity of the supply chain to the change of internal variable $r$ ($dr$)

6.5 Interpretation of the bifurcation analysis for the practice

Bifurcation diagrams are of necessary importance as they can be used to define the ranges of the internal parameters of the supply chain in which the supply chain is in stable state. This analysis also helps to identify what type of states presented in the supply chain within the window range of the parameter. Indeed, in this work, various states of the supply chain were observed raging from regular to chaotic states. These states were observed when monitoring the internal parameters (for example $\sigma$ and $r$ ) of the supply chain. Therefore, the bifurcation analysis can help to discover various states towards the achievement of the cancelation of the effect of uncertainties. This analysis can also be used to control and cancel the effects due to external perturbations.
6.5.1 Significance at strategic level

The bifurcation analysis is particularly helpful at the stage of the strategic level to plan the SCN. Some of the key benefits of the bifurcation analysis at the strategic level are summarized as follows:

- The key parameter space identification helps in designing the stable supply chain.
- Bifurcation analysis aids in drafting the cooperation and coordination contracts.
- Analysis of the possible states of the supply chain helps in re-engineering the supply chain.
- It aids in the decisions related to the warehouse and inventory replenishment policies.

6.5.2 Significance at tactical and operational level

The decisions made at the tactical and operational level are more in the direction of controlling and improving the operational efficiency of the supply chain. The decision managers at this level are often expected to deal with the uncertainties. Bifurcation analysis can help in mitigating the effects of uncertainties without affecting the whole supply chain performance. The managers or the decision support systems can use the information from the bifurcation analysis in adjusting the parameters. Bifurcation analysis also helps in understanding the effects caused by the external infertilities by identifying the current system state. In addition to this, it also improves the coordination and trust between the players. If every player in the supply chain uses the information from the bifurcation analysis to mitigate the effects of external uncertainties, the chances are high that the system is in stable state.

6.6 Summary

In this chapter, the importance of identifying/analyzing the nonlinear dynamics in SCNs’ is discussed. A theoretical model of the SCN with manufacturer, distributor and retailer is presented and modeling of the supply chain using coupled oscillatory systems based modeling method is explained in detail. The requirements and the assumptions in the modeling process are described. Mathematical formulation of the dynamics between the three players in the SCN by identifying the important parameters and their relations is carried out step-by-step. The example supply chain is defined in this chapter by assigning the parameter values for the internal parameters. The analysis of the nonlinear dynamics presented in the example SCN under the influence of the external uncertainties is performed with phase space analysis. The investigation
into the dynamics of SCN with different external perturbations is conducted to analyze the SCN behavior. This analysis reveals that the SCN subjected to external uncertainties undergoes both saturation state and chaotic state. In order to avoid or cancel these states, the players in the SCN respond by increasing the inventories and changing the coefficients of information distortion, customer satisfaction rates. However, this change in the parameters that are under their control discrepantly can effect the global supply chain performance. From the theory of dynamic systems, it is proven that the change in tiny/small variations of the parameters can again lead the system to exhibit various dynamic states. Bifurcation analysis can help to find the "islands of stability" for these key parameters which can be used to mitigate the effect of external uncertainties. In this chapter, the bifurcation analysis is performed on the key internal parameters to identify the "islands of stability" for these key parameters.
Chapter 7

Nonlinear control and synchronization of supply chains

7.1 Basics of nonlinear control and synchronization

The subject of nonlinear control deals with the analysis and the design of nonlinear control systems, i.e. control systems containing at least one nonlinear component. In the analysis of the system, a nonlinear closed-loop system is assumed to have been designed, and we wish to determine the characteristics of the system’s behavior [91]. In the design, we are given a nonlinear system to be controlled and some specifications of closed-loop system behavior, and our task is to construct a controller so that the closed loop system meets the desired characteristics [95]. In practice, of course, the issues of design and analysis are intertwined, because the design of a nonlinear control system usually involves an iterative process of analysis and design [95]. Physical systems are inherently nonlinear. Thus, all control systems are nonlinear to a certain extent. Nonlinear control systems can be described by nonlinear ordinary differential equations (ODE). However, if the operating range of a control system is small, and if the involved nonlinearities are smooth, then the control system may be reasonably approximated by a linearized system, whose dynamics is described by a set of linear differential equations.

7.1.1 Comparison of linear and nonlinear control theory

Linear control theory has been predominantly concerned with the study of linear time invariant (LTI) control systems, of the form with \( \mathbf{x} \) being a vector of states and \( \mathbf{A} \) being the system matrix. LTI systems have quite simple properties, such as [91, 96]:

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A linear system has a unique equilibrium point if $A$ is nonsingular.

The equilibrium point is stable if all eigenvalues of $A$ have negative real parts, regardless of initial conditions.

The transient response of a linear system is composed of the natural modes of the system, and the general solution can be solved analytically.

In the presence of an external input $u(t)$, the system response has a number of interesting properties such as satisfaction of the principle of superposition, bounded-input bounded-output stability in the presence of $u$ and a sinusoidal input leads to a sinusoidal output of the same frequency.

The behavior of nonlinear systems, however, is much more complex. Due to the lack of linearity and of the associated superposition property, nonlinear systems respond to external inputs quite differently from linear systems.

### 7.1.2 Necessity of nonlinear control and synchronization

Nonlinear control is necessitated by many requirements for improvement of the systems and to especially to cope up with uncertainties within the model and from external uncertainties [91].

**Improvement of existing control systems:** Linear control methods rely on the key assumption of small range operation for the linear model to be valid. As the nonlinearities in the system cannot be compensated properly, poor or unstable performance is likely to be the result from the linear controller when the required operation range is large. Nonlinear controllers on the other hand, may handle the nonlinearities in large range operation directly [96].

**Analysis of hard nonlinearities:** Another assumption of linear control is that the system model is indeed linearizable. However, in control systems there are many nonlinearities whose nature does not allow linear approximation. These so-called ”hard nonlinearities” include saturation, dead-zones, backlash, and hysteresis, are often found in control engineering. Their effects cannot be derived from linear methods, nonlinear analysis techniques must be developed to predict a system’s performance in the presence of these inherent nonlinearities [96]. Because such nonlinearities frequently cause undesirable behavior of the control systems, such as instabilities, their effects must be predicted and properly compensated. Such nonlinearities are prevalent in business systems.
Dealing with uncertainties: In designing linear controllers, it is usually necessary to assume that the parameters of the system model are reasonably well known. However, many control problems involve uncertainties in the model parameters. This may be due to a slow time variation of the parameters (e.g. rate of information distortion under market fluctuations), or to an abrupt change in parameters due to sudden factory breakdowns or transportation problems. A linear controller based on inaccurate or obsolete values of the model parameters may exhibit significant performance degradation or even instability (saturation or chaos). Nonlinearities can be intentionally introduced into the controller part of a control system so that model uncertainties can be tolerated [96].

Design Simplicity: Good nonlinear control designs may be simpler and more intuitive than their linear counterparts. This, a priori paradoxical result comes from the fact that nonlinear controller designs are often deeply rooted in the physics of the plants.

7.2 Analysis and control of nonlinear dynamic systems

The study of these nonlinear analysis techniques is important for a number of reasons such as:

- Theoretical analysis is usually the least expensive way of exploring a system’s characteristics.

- Simulation, though very important in nonlinear control, has to be guided by theory. Blind simulation of nonlinear systems is likely to produce few results or misleading results. This is especially true, given the great variety of behavior that nonlinear systems can exhibit, depending on initial conditions and inputs.

- The design of nonlinear controllers is always based on analysis techniques. Since design methods are usually based on analysis methods, it is almost impossible to master the design methods without first studying the analysis tools.

Furthermore, analysis tools also allow us to assess control designs after they have been made. In case of inadequate performance, they may also suggest directions of modifying the control designs. Many methods of nonlinear control system analysis have been proposed.

Phase plane analysis (detailed description in section 5.2) is a graphical method for studying second order systems. Phase plane analysis has a number of useful properties but the main disadvantage of the method is that it is restricted to second order (or first order) systems,
because the graphical study of higher order systems is computationally and geometrically complex [96].

The other popular method used to analyze the nonlinear dynamic systems is based on the Lyapunov theory [95, 96]. Basic Lyapunov theory comprises of two methods introduced by Lyapunov, the indirect method and the direct method. The indirect method, or linearization method, states that the stability properties of a nonlinear system in the close vicinity of an equilibrium point are essentially the same as those of its linearized approximation [96]. The method serves as the theoretical justification for using linear control for physical systems, which are always inherently nonlinear. The direct method is a powerful tool for nonlinear system analysis, and therefore the so-called Lyapunov analysis often actually refers to the direct method.

The direct method is a generalization of the energy concepts associated with a mechanical system: the motion of a mechanical system is stable if its total mechanical energy decreases all the time [91]. In using the direct method to analyze the stability of a nonlinear system, the idea is to construct a scalar energy-like function (a Lyapunov function) for the system, and to see whether it decreases or not [91]. The power of this method comes from its generality: it is applicable to all kinds of control systems, be they time-varying or time-invariant, finite dimensional or infinite dimensional [91]. Conversely, the limitation of the method lies in the fact that it is often difficult to find a Lyapunov function for a given system [91]. Although Lyapunov’s direct method is originally a method of stability analysis, it can be used for other problems in nonlinear control. One important application is the design of nonlinear controllers. The idea is to somehow formulate a scalar positive function of the system states, and then choose a control law to make this function decrease. A nonlinear control system thus designed will be guaranteed to be stable [91]. Such a design approach has been used to solve many complex design problems, e.g., in robotics and adaptive control. The direct method can also be used to estimate the performance of a control system and to study its robustness.

The describing function method is an approximate technique for studying nonlinear systems. The basic idea of the method is to approximate the nonlinear components in nonlinear control systems by linear "equivalents", and then use frequency domain techniques to analyze the resulting systems. Unlike the phase plane method, it is not restricted to second-order systems. Unlike Lyapunov methods, whose applicability to a specific system hinges on the success of a trial-and-error search for a Lyapunov function, its application is straightforward for nonlinear systems satisfying some easy-to-check conditions. The method is mainly used to predict limit cycles in nonlinear systems [91].
7.2. ANALYSIS AND CONTROL OF NONLINEAR DYNAMIC SYSTEMS

7.2.1 Controlling and synchronization of nonlinear dynamic systems

Linear control methods rely on the key assumption of small range operation for the linear model to be valid. When the required operation range is large, a linear controller is likely to perform very poorly or to be unstable, because the nonlinearities in the system cannot be properly compensated for. Nonlinear controllers, on the other hand, may handle the nonlinearities in large range operation directly.

Sliding mode control

Sliding mode control is a standard approach to tackle the parametric and modeling uncertainties of a nonlinear system. For sliding mode controller, Lyapunov stability method is applied to keep the nonlinear system under control [97]. Nonlinear system model imprecision may come from actual uncertainty about the system (e.g. unknown system parameters), or from the purposeful choice of a simplified representation of the system’s dynamics [98]. Modeling inaccuracies can be classified into two major kinds: structured (or parametric) uncertainties and unstructured uncertainties (or unmodeled dynamics). The first kind corresponds to inaccuracies on the terms actually included in the model, while the second kind corresponds to inaccuracies on the system order [97, 98].

Modeling inaccuracies can have strong adverse effects on nonlinear control systems. One of the most important approaches to dealing with model uncertainty are robust control. The typical structure of a robust controller is composed of a nominal part, similar to a feedback control law, and additional terms aimed at dealing with model uncertainty. Sliding mode control is an important robust control approach. For the class of systems to which it applies, sliding mode controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision. On the other hand, by allowing the tradeoffs between modeling and performance to be quantified in a simple fashion, it can illuminate the whole design process. The nominal part in the controller is not a continuous function, it switches from one structure to another based on its current position in the state space [97, 98]. For this reason it also called as the variable structure control (VSC).

The purpose of the switching control law is to drive the nonlinear system’s state trajectory onto a prespecified (user-chosen) surface in the state space and to maintain the systems’s state trajectory on this surface for subsequent time. The surface is called a switching surface.
When the system state trajectory is "above" the surface, a feedback path has one gain and a different gain if the trajectory drops "below" the surface. This surface defines the rule for proper switching. This surface is also called a sliding surface (sliding manifold). Ideally, once intercepted, the switched control maintains the system's state trajectory on the surface for all subsequent time and the system's state trajectory slides along this surface. The most important task is to design a switched control that will drive the system state to the switching surface and maintain it on the surface upon interception [97, 98]. A Lyapunov approach is used to characterize this task. A generalized Lyapunov function, that characterizes the motion of the state trajectory to the sliding surface, is defined in terms of the surface. For each chosen switched control structure, one chooses the "gains" so that the derivative of this Lyapunov function is negative definite, thus guaranteeing motion of the state trajectory to the surface. After proper design of the surface, a switched controller is constructed so that the tangent vectors of the state trajectory point towards the surface such that the state is driven to and maintained on the sliding surface. Such controllers result in discontinuous closed-loop systems. The more finer details on mathematical modeling of the sliding mode controller can be seen in [97, 98].

Adaptive control

Most current techniques for designing control systems are based on a good understanding of the system under study and its environment. However, in a number of instances, the system to be controlled is too complex and the basic physical processes in it are not fully understood. Control design techniques then need to be augmented with an identification technique aimed at obtaining a progressively better understanding of the system to be controlled. It is thus intuitive to aggregate system identification and control. Often, the two steps will be taken separately. If the system identification is recursive, that is the system model is periodically updated on the basis of previous estimates and new data identification and control may be performed concurrently [99].

The parameters of the controller are adjusted during the operation of the system as the amount of data available for system identification increases. For a number of simple PID (proportional + integral + derivative) controllers in process control, this is often done manually. However, when the number of parameters is larger than three or four, and they vary with time, automatic adjustment is needed. In the literature of adaptive control [99, 100, 101], the on-line parameter estimator has often been referred to as the adaptive law, update law, or adjustment mechanism. The design of the adaptive law is crucial for the stability properties of the adaptive controller. The adaptive law introduces a multiplicative nonlinearity that makes the closed-loop
7.3. APPROPRIATE CONTROL AND SYNCHRONIZATION TYPES IN SUPPLY CHAINS

system nonlinear and often time varying [100]. Because of this, the analysis and understanding of the stability and robustness of adaptive control schemes are more challenging. One of the methods to design adaptive laws is based on Lyapunov design. This method of developing adaptive laws is based on the direct method of Lyapunov and its relationship with positive real functions. In this approach, the problem of designing an adaptive law is formulated as a stability problem where the differential equation of the adaptive law is chosen so that certain stability conditions based on Lyapunov theory are satisfied. The design of adaptive laws using Lyapunov’s direct method are more detailed in these works [101].

Active control

Recently, a number of control strategies to suppress bifurcations and chaos in nonlinear systems have been proposed in the literature [102, 103]. A general framework for the control of local bifurcations, saturation and chaos, in non autonomous systems is using feedback strategies. It is shown [102] that a system exhibiting chaos can be driven to a desired periodic motion by designing a combination of feed forward controller and a time-varying controller. The active control synchronization follows the phenomena that the trajectories of the of the two systems, reference system and disturbed system, should be identical even if the initial conditions are different or the disturbed system is influenced by internal/external perturbations [102]. This motivated in considering the active control theory based approach in controlling the effects caused by external uncertainties on SCN.

Recently, Bai and Lonngren proposed active control techniques to synchronize two autonomous systems [102]. In this method of active control, the model of the slave system consists of control input i.e the externally perturbed system is given with the input of the control unit. The control functions are determined by subtracting the model of slave system from the master system, reference system, resulting in the error dynamics in the convergence of two systems. Now, the control functions are chosen such that the resulting error dynamics consists of only the error variables. The mathematical modeling of the controller is explained in section 7.4 in detail.

7.3 Appropriate control and synchronization types in supply chains

In this section we briefly introduce the importance of synchronization and present the modern methods/tools used to deal with it. When organizations synchronize supply chains and have
real-time access to data, they enjoy several competitive advantages:

- Anomalies and exceptions are identified early and the data for intelligent response is immediately available. This greatly minimizes the bullwhip effect and saves downstream partners and customers from needless activity.

- Back-end processing of change orders, modified invoices, and updates to inventory systems can happen as decisions are being made [55].

- Visibility into partner systems makes planning easier and enables managers to see opportunities that were not apparent before. This visibility also enables supply chain partners to collaborate more effectively. This cooperation is essential when problem resolution requires the coordination of several supply chain partners.

- Easier integration and expansion. Since the synchronization and data passing mechanisms are not based on packages from supply chain software vendors or ERP firms, the software is optimized for all players. Packages can be changed without disrupting the infrastructure and new partners can be added easily.

To date, the majority of supply chain efficiencies has come from improvements within the four walls of each individual company [56]. Certainly, there has been collaboration between trading partners in terms of improved communication (EDI and current internet based web information exchanges), better information (point of sale data and the CPFR initiatives), and a general willingness to work more closely together. Even though these information sharing methodologies have provided synchronization to some extent, still there exists some discrepancies as detailed in section 2.5. Apart from all these data discrepancies, there may be other disturbances which occur due to external perturbations. There are different methodologies applied to study these discrepancies and to quantify their effects. In this thesis we focus on two major synchronization types applied in supply chain networks based on the supply chain model [55] namely:

- Complete Synchronization: the cycle time at an upstream stage equal to cycle time of the next downstream stage.

- Partial synchronization: the cycle time at an upstream stage is an integer multiple of the cycle time of the next downstream stage.

7.3.1 Complete synchronization

Complete synchronization enables the supply chain to react quickly to changes in demand and in product design. This type of synchronization is particularly suitable in just-in-time supply
7.3. APPROPRIATE CONTROL AND SYNCHRONIZATION TYPES IN SUPPLY CHAINS

chain networks [56]. To achieve the complete synchronization the complete SCN should be
totally integrated. Many retailers and manufacturers have looked to the internet to provide
a cost-effective avenue for improving the data deficiencies present in today’s supply chains.
One of the main ways to decrease the amount of inconsistent item information being passed
among companies is to automate many of the item maintenance functions, such as data changes
and new item introductions, performed by manufacturers. Synchronization tools automate
processes that are traditionally conducted through excessive paperwork, such as informing
trading partners of changes to product information. This synchronization method affects the
registry and life cycle management of synchronized products, locations and trading capabilities
[48]. Data synchronization involves a strategic combination of technologies, business strategies
and implementation of industry standard in messaging that ties together the information
management systems of all parties participating in a given supply chain [55].

The modern synchronization tools provide automation of the paper work but do not
concentrate on what happen if the given data is slightly changed accidentally. Effective data
synchronization first involves making information visible within a secured environment, which
is again a case that we have discussed in chapter 3.

7.3.2 Partial synchronization

Partial synchronization is achieved through the controller item [67]. Apart from the data
synchronization as explained in complete synchronization, a controller item is developed to
mitigate the effect caused due to time lag as shown in figure 7.1. In these synchronization major
emphasis is placed in analyzing the system and modeling the effect of lack of synchronization
and quantifying it. The modeling and quantification of the effect caused by the time lag,
information discrepancy, and individual objectives helps in designing the controller/synchronizer
element. This controller item can be unique for each entity or supply chain as a whole. Partial
synchronization is also achieved indirectly by quantifying the region where uncertainties ex-
ists. After identification of uncertain regions they try to avoid or extra careful in that region [67].

In practice, management of commercial SCN is a challenging issue as diverse pre-defined
‘data’ thresholds or requirements within them should be currently stabilized with regard to
the existence of some uncontrollable effects due to both internal and external fluctuations,
as discussed in chapter 3, to which the SCN may be subjected to. The dynamic character
of the information flows within the commercial supply chains is another factor which makes
it very difficult to stabilize the overall behavior of the whole supply chain. Therefore, the

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instability of the supply chain may be directly perceived as the modification of data thresholds or pre-defined requirements based on the uncertainties discussed in chapter 3. As we have seen in chapter 6, the effects of the external perturbation on the stability of the system by forcing it into saturation and chaotic states occasionally, in the next section we see the design of a controller item that minimizes the effects and brings back the system into regular state.

7.4 Design of a synchronization controller for a supply chain

We have seen in the chapter 6, the SCNs are very unstable, saturated or driven into chaos, when dealing with the uncertainties. The causes of instability of the supply chain can be broadly classified into two categories. The first cause is the dynamical and non-linear character of the motions (i.e. material/products flow and information exchanges) between different entities in supply chains. The second cause originates from the effects of both external and internal perturbations to which the supply chain is subjected to.

An optimal management of the information flows within the supply chains may be of high importance in order to alleviate the effects leading to negative consequences on the flows within the supply chains. This could be achieved through an active control mechanism which is based on a current comparison of the dynamical data within the supply chains with the pre-defined data fixed by the requirements of the supply chains as shown in figure 6.2. Here, an automatic or active control of the flows within the supply chains should be able to detect changes in the
flows and act accordingly/consequently (by undertaking a given action), in order to alleviate the undesirable effects and therefore stabilize the system behavior that has been perturbed. The achievement of synchronization is observed when the action undertaken has allowed the recovery of the original behavior (eventually thresholds or reference requirements) of the supply chain. The schematic description of the active synchronization of a supply chain is illustrated in figure 7.1.

### 7.4.1 Synchronization controller design using active control method

The system of ordinary differential equations that has been used to model the dynamic of the supply chain is written as shown in Eq. 6.5. The reference supply chain network model is defined in section 6.3 by fixing the key parameter values. We have also defined the externally perturbed system in Eq. 6.6. We call the reference supply chain model as master system and the perturbed system as slave system. In Eq. 6.6, we have introduced the external fluctuation as $d_1$, $d_2$ and $d_3$. These external fluctuations are to be nullified. In order to ascertain the control functions, we subtract Eq. 6.6 from Eq. 6.5. It is convenient to define the differences between the reference system and perturbed system using Eq. 7.1.

\[
\begin{align*}
    e_x &= \dot{x} - x \\
    e_y &= \dot{y} - y \\
    e_z &= \dot{z} - z
\end{align*}
\] (7.1)

Using this notation, the resulting error system in continuous form is derived in Eq. 7.2:

\[
\begin{align*}
    \dot{e}_x &= \sigma(e_y - e_x) + d_1 \\
    \dot{e}_y &= r e_x - e_y - \dot{x}z + xz + d_2 \\
    \dot{e}_z &= \dot{x}y - xy - be_z + d_3
\end{align*}
\] (7.2)

Now we define the active control functions $V_1$, $V_2$ and $V_3$ as shown in Eq. 7.3:

\[
\begin{align*}
    V_1 &= d_1 \\
    V_2 &= \dot{x}z - xz + d_2 \\
    V_3 &= -\dot{x}y + xy + d_3
\end{align*}
\] (7.3)
This leads to

\[
\begin{align*}
\dot{x} &= \sigma(e_y - e_x) + V_1 \\
\dot{y} &= re_x - e_y + V_2 \\
\dot{z} &= -be_z + V_3
\end{align*}
\]  

(7.4)

Eq. 7.4 describes the error dynamics and can be considered in terms of a control problem where the system to be controlled is a linear system with a control input \( V_1, V_2 \) and \( V_3 \) as functions of \( e_x, e_y \) and \( e_z \). As long as these feedbacks stabilize the system, \( e_x, e_y \) and \( e_z \) converge to zero as time \( t \) goes to infinity. This implies that two systems are synchronized with feedback control. There are many possible choices for the control \( V_1, V_2 \) and \( V_3 \). We choose:

\[
\begin{pmatrix}
V_1 \\
V_2 \\
V_3
\end{pmatrix} = A
\begin{pmatrix}
e_x \\
e_y \\
e_z
\end{pmatrix}
\]

Where \( A \) is a 3X3 constant matrix. For proper choice of the elements of the matrix \( A \), the feedback system must have all of the eigenvalues with negative real parts.

The active control algorithm considers the effects of external perturbations and adjusts the values of the internal parameters \( (\sigma, r, b) \) of the three-echelon supply chain by tiny variations \( d\sigma, dr \) and \( db \). The variation of each internal parameter is performed in well defined ranges (or windows) of variation. Performing the parameters variations in these ranges is necessary as we cannot vary the parameters beyond the realistic scenario. A threshold error is fixed (which is less than approximately 0.02) under which full alleviation of the effects due to external perturbations is supposed to be effective; this leads to the achievement of synchronization, which results in the recovery of the behavior of the reference system.

### 7.4.2 Alleviation of the effects caused by uncertainties on the example supply chain

For the structure shown in figure 6.4 (see section 6.3), which shows the occurrence of the well-known bullwhip and chaotic effects in the three-level supply chain by displaying discernible patterns. The regulation process has been exploited to adjust the internal parameters values in order to achieve synchronization, i.e. the full cancelation of the effects due to perturbations. The corresponding values obtained from the active control processes for the achievement of the regulation process are \( d\sigma = 35 \), \( dr = 15 \) and \( db = 0.09 \). The phase space portrait displaying the trajectory of the system after the control process is shown in figure 7.2. Indeed, below
the precision of 2% when compared with the reference model in figure 6.2. Figure 7.2 shows attractors similar to those of the reference supply chain model.

Figure 7.2: Alleviation of the chaotic effect caused by external perturbation with active synchronization controller

Like wise in order to alleviate the effect of saturation due to external perturbations as shown in section 6.3, we performed the active control theory method as explained before by adjusting the internal parameters of the SCN. The appropriate values of the internal parameters to alleviate the effects due to external perturbations are $d\sigma = 4$, $dr = 2$ and $db = 1$. The phase space portrait displaying the trajectory of the system after the

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control process is shown in figure 7.3. Indeed this process shows that tiny variations of the internal parameters of the supply chain lead to the achievement of synchronization. This shows the change of the state of the system from the saturation state (figure 6.3) to a regular state (figure 7.3) which is similar to the state of the reference supply chain (figure 6.2).

7.5 Summary

In this chapter the basics of nonlinear control and synchronization are introduced. The differences between the linear and nonlinear control are discussed in detail before identifying the necessity of the nonlinear control and synchronization. Nonlinear control is necessitated by many requirements for improvement of the systems and to especially to cope up with uncertainties within the model and from external uncertainties. Nonlinear dynamic analysis tools also allow us to assess control designs after they have been made. In case of inadequate performance, they may also suggest directions of modifying the control designs. Many methods of nonlinear control system analysis have been proposed. The two important analysis tools which aid in the process of controller design namely phase space analysis and Lyapunov theory based methods are explained in detail. Linear control methods rely on the key assumption of small range operation for the linear model to be valid. Nonlinear controllers, on the other hand, may handle the nonlinearities in large range operation directly. The three important nonlinear control methods namely sliding mode control, adaptive control and active control methods are explained in detail before detailing the method adopted in this thesis. The active control synchronization follows the phenomena that the trajectories of the of the two systems, reference system and disturbed system, should be identical even if the initial conditions are different or the disturbed system is influenced by internal/external perturbations. By considering this, in this thesis active control approach is used to design the controller to mitigate the effects caused by external uncertainties. The controller design for the supply chain model proposed in chapter 6 is explained in this chapter.
Chapter 8

Nonlinear dynamics in short life cycle products supply chain - a case study

8.1 Introduction to short life cycle products supply chain

The life cycle of the product is an important element in designing and deploying an effective supply chain. Life cycle of product can be defined as \[29\] \textit{the length of time from the stage of products introduction into the specific market place, through the growth and maturation till the products decline in the market place}. The life cycle of the product stretches into various stages such as product development, introduction into market and growth of the product in the market. The shape of the product life cycle curve depends on the product type and the stages it undergoes as shown in the figure 8.1.

The duration of the product’s life cycle can be few months and can stretch over number of years like oil SCNs. The globalization of competition among the contemporary industries is increasingly shortening the product life cycles. According to P. O’Connell, President of Operations Concepts Incorporation, a standard product life cycle lasts up to two years [104]. In the opinion of M.L. Fisher, a short life cycle product varies from three months to one year and longer product life cycle is over two years [105]. This view is also supported by H. Peck and M. Christopher, who argue that a short life cycle product lasts up to 12 months [106]. In this thesis, we consider the short life cycle as a life cycle of the product from its launch till the decline in the market stretches over 60-80 weeks.
The short life cycle is the main characteristic of the following products categories [10]:

- Information technology oriented products such as personal computers, printers, software solutions and mobile phones.
- Consumer goods such as fashion goods, electronic equipment, toys, jewelry, books and CDs.

The shortening in the life cycle of the products is mainly connected to the availability of many choices to the customer due to the globalization, permanent development of technology, innovative character of the products, and many sources of customer response such as feedback responses via internet, reviews from experts and forum discussions leading to the improvement of products continuously.

### 8.1.1 Challenges with short life cycle products

The short life cycle product supply chains heavily depends on highly accurate forecast at many stages. The traditional four-step life cycle of a product, shown in figure 8.1, passes several quadrants of variance, which influence the supply chain. The short life cycle of a product can also be a result of less predictable variance of customers demand. The specific stages of the product life cycle can be referred to both the demand uncertainty and its variability (figure 8.2).

A. Bhattacharya, et al [107], classified the disturbance of the product flow due to the uncertainties in demand into several categories. The two key categories that are specifically important in the context of the short life cycle products’ supply chain are namely design and

![Figure 8.1: Different stages in life cycle of a product](image-url)
8.1. INTRODUCTION TO SHORT LIFE CYCLE PRODUCTS SUPPLY CHAIN

Changes in customers’ demand
Variability of customers’ demand
Uncertainty of customers’ demand volume/design turbulence

INTRODUCTION
Little product variety and volume, Innovative product development

GROWTH
Product variety and volume increases, Innovative product development

MATURITY
Product variety increases, Product volumes decreases, Substitute products development

DECLINE
Product variety increases, Product volumes declines, Substitute products development

SUPPLY CHAIN

Figure 8.2: specific stages of the product life cycle effected by the uncertainty (taken and adapted from [10])

volume. The generation of uncertainty due to the design relates to disturbances caused in the production flow by changes in the product design. This kind of uncertainty is more when the manufactured products go through numerous design changes within their life cycle. This design related uncertainty is more in the mass customized products as the customer preferences and products from competitors force the design changes. Because products in mass customized markets have short life cycles, it is difficult to buffer variations in demand even when they are expected by the inventories.

The volume of the production is the other key source for the generation of uncertainty in the short life cycle products’ demand prediction. During the introduction phase of the life cycle, little volume and variety is produced by manufacturers. The volume sizes increases in the growth phase. The growth phase is the challenging phase for the SCN design as it is the time to get organized, formalized and to structure the SCN to cope with the market demand. The operational efficiency of the supply chain is important to satisfy the customer demand. During maturity phase, manufacturers introduce the variety in a response to decelerating growth within the existing main product line and the supply chain has to be adjusted accordingly. In the decline stage, variety continues to increase as market segments scatter into micro markets, while volume declines [108]. This variation in the production volume generates the uncertainty in predicting the customer demand.

Apart from the design and volume, the observed changes in specific industries (for example
the trucking industry, labor market, raw material industry) influence the supply chain, forcing an organization to be more responsive, flexible and, finally, agile. The sourcing of the suppliers required for manufacturing the new product is also a challenging task for short life cycle products as the new products demand can’t be predicted or in other words the demand is of stochastic nature. Earlier research work assumed that the procurement decision had to be made before the realization of demand.

The other key challenge is the inventory management at distributors and retailers. Recent work that focuses on retailer’s or distributor’s inventory management problem for short life cycle product includes Bradford and Sugrue [109] and Fisher et al. [105]. The inventory management at retailer and distributors is particularly important during the maturity and decline phase of the supply chain.

Supply chain organization must be able to cope with the effects of demand instability and learn to adjust to the permanent changes in customers’ needs and requirements. In other words, organizations have to adopt modern initiatives (methods of management and supporting technical tools) which attempt to remove some of the causes of customers’ demand variability and uncertainty, and thus to reduce investments and time-to-solutions.

8.1.2 Case study - Virtual toy supply chain network

The SCN design and operational requirement differ considerably depending upon the industry. Each industry has unique requirement that suits them as the characteristics of the players, relationship among the players in the SCN is different. Toy industry is one of the typical example for the short life cycle product supply chain. It is one of the key industry which demands high level creativity and innovation to attract the customers. For centuries, toys are the main source of entertainment to children. In recent decades toys are also used to educate the children, used as emulators, nurturers, friends, etc. The modern toys can be divided into three categories namely [110]:

- Traditional toys: dolls, family games, card/dice games, vehicles etc.

- Virtual toys: gadget based toys such as Tamagotchi™ and Giga pets, web based toys such as Mystic Island or tiki village in Mrtiki, Anatheria in Anatheria.

- Software toys: computer simulation based toys and games such as Sim City 2000 and Second Life.
The growth of the traditional toys sales is slowly going down while the growth in the sales of virtual and software toys is steadily increasing [110].

The toys SCN consists of the component suppliers, manufacturers, distributors or wholesalers, retailers and customers. Even though the customers for the toys are the children and the parents or grandparents (elders) are paying the money for the toys. The retailers play an important role in the toys SCN. The retailers in the toys SCN can be classified into two categories namely: “just-in-time” (JIT) or “pull” retailers and “mixed model” retailers. The pull retailers are the specialist toy sellers located in sub-urban and selling large variety of toys at either low or medium prices. Their demands are mainly pulled and the pushed demand is only around 20%. The pushed demand is either to introduce the new toy or to prepare for the season such as Christmas, Easter and New year. These retailers do not like to host obsolete inventory and don’t like the financial commitment. The other type of retailers, “mixed model”, sell many other types of merchandize together with the toys. These retailers are usually the supermarkets, department stores and hypermarkets. These retailers keep only limited variety at medium or promotional prices. The major component of their demand is always pushed. This type of distributors own regional distribution centers to penetrate into the market.

Virtual toys relatively have less lifecycle than the traditional toys as the innovative and creative scope of these toys is high and the technological advancements directly reflect in these toys. Virtual toys manufacturers have to deal with the obsolete inventory, sudden drop in the rate of sales compared with other industries. These issues answers the typical questions listed in Chapter 1 in selecting the case study to validate the applicability of the methods/tools from the nonlinear dynamic systems theory to SCM.

### 8.2 The Tamagotchi™ supply chain network

Bandai, a Japanese toy manufacturing company, introduced the Tamagotchi™ to the Japanese market during the fourth quarter of 1996. Bandai founded in 1900 is the one of the largest producer of toys. Bandai has toys licence to many popular characters such as Gundam, Power rangers, the super sentai and Ben 10. Bandai classified their products into 8 categories: vending machine products, apparel, video games and general toys, toys for boys, toys for girls, snacks and others [111]. Tamagotchi™ is categorized in video games and general toys. Tamagotchi™ is an egg shaped gadget, figure 8.3, and a first version of the virtual pet. Tamagotchis created a different class in virtual pets by acquiring the physically in the form of gadget which can
be carried in the pocket or hanged in the neck. The goal of this game is to raise the pet, Tamagochi™, by feeding, taking care of its health and all other issues that come in raising the pets. With this type of continuous interaction between the human and the toy, Bandai has created the virtual love. The traditional toy and child/consumer relationship is changed significantly with the introduction of virtual love. Bandai originally developed this concept as a method to train/educate and develop interest in the Japanese girls on raising children.

Bandai estimated that Tamagotchi™ has the potential to be a successful product in the market even without mass media advertisements. They believed word of mouth among the target group, high school girls, is enough to popularize the toy. However, the popularity of the tamagotchis is not just limited to high school girls. The popularity spread to the primary school boys and girls, office secretaries and male executives also. Bandai estimated to sell 300 thousand units before the end of the year 1996 in the Japanese domestic market. Bandai spokeswoman Tomio Motofu said that the firm sold 450 thousand tamagotchis in just 38 days since the November 23rd launch in 1996 [112]. The widespread range of the target group increased the tamagotchis sales by 4 million by the end of the first quarter of 1997. Bandai also stated to introduce the product into the European and North American markets, largest markets in the world for toys, in middle of second quarter of 1997 [112]. By end of the third quarter the overseas sales exceeded 2.4 million units. This sudden boom in demand for the units outpaced the ability of the Bandai in manufacturing the units.

The inability of the Bandai in meeting the demand produced many problems in the society.
8.2. THE TAMAGOTCHI™ SUPPLY CHAIN NETWORK

The crimes of robberies, assaults to acquire the toys, quarrels among the school children, long queues lasting over days at the toy stores that had no inventory or smaller inventory at department stores, and greediness among the school children are only a few problems to name. These social problems created the shortage game by the customers, complaints from the retailers and copy problems. Bandai received about 5000 complaints over phone per day about shortages from retailers [112]. This shortage also forced alternate retail markets to emerge to encash the demand. The reselling by the consumers through alternate channels increased and the price of tamagotchis in these channels is more than 50 times its retail price of 1,980 yen ($16) [113].

Moreover to keep up with the demand, Badai made a decision in July 1997 to increase the manufacturing capacity to 2 million units per month [112]. Even though the managers at Bandai realized that this increase in manufacturing capacity results in excess inventories and excess capacity they are forced to do this as the dissatisfaction with target group customers effects their business with other categories of toys. Interestingly, after expanding the manufacturing capacity, Bandai faced sharp decline in the demand as many retailers canceled duplicate orders and the capricious demand generated by the shortage game is reduced.

Even though the Tamagotchi™ product is highly successful and a revolution in the virtual pet toys industry, Bandai announced the loss of 16 billion yen in after-tax for the fiscal year of 1998 [113]. The losses are mainly due to the obsolete inventories at retailers and manufacturer. This case illustrate the need for efficient tools to estimate the over all dynamics of the SCN. This detailed case of the Tamagotchi™ with the clearly defined and documented dynamics between the players provide the best case to study the usability of the methods/tools from the nonlinear dynamic systems in efficiently managing the SCN. Before presenting the application of the tools from nonlinear dynamic systems, in order to illustrate what happened at the Bandai, in this thesis we use the systems dynamics based modeling and simulation of the Tamagotchi™ SCN.

8.2.1 Modeling using system dynamics approach

Higuchi [113] provided the conceptual frame work of the Tamagotchi™ SCN detailing the dynamics involved in managing SCN. In this thesis we adapt the dynamics provided by Higuchi [113] to analyze the the adaptability of the methods and tools from nonlinear dynamics to the real world supply chains such as the Tamagotchi™ SCN. Tamagotchi™ SCN model is also divided into three levels, the manufacturer, retail and customer market as described in our theoretical three level model of the supply chain in Chapter 6 (figure 6.1). Figure 8.4
describes the conceptual framework of the Tamagotchi™ SCN. The demand at the market level is the cumulative demand generated by the new customers, the demand generated due to the shortage of supply, and the customers who brought the product buying again. The demand generated by the shortage of the products is called as phantom demand. In order to estimate the penetration of the product into the market, the popular logistic curve function is applied. Logistic curve function is popularly used in modeling the epidemic propagation models. The Tamagotchi™ model is also similar to the epidemic model in many aspects such as penetration in target community and the effect of available resources in the penetration into the market. The logistic curve is given by the ordinary differential equation given in Eq. 8.1.

\[
\dot{x}_t = \alpha x_t (K - x_t) \tag{8.1}
\]

Where \(x_t\) is the cumulative number of people who purchased by the end of time \(t\).
\(\dot{x}_t\) is the derivative of \(x_t\).
The parameter \(\alpha\) is a small number that controls the diffusion speed, the speed with which the product penetrates into the market, where bigger values are associated with faster diffusion.
\(K\) is the theoretical upper limit of the number of purchases.

This logistic curve is especially suitable for this model due to the fact that, if the shortage occurs in the market, the company losses their potential customers and the value of \(K\) becomes smaller. The theoretical upper limit is set to 15% of the Japan population as the initial target group is only high school going girls. However to know how the product is penetrating into target group setting up \(\alpha\) is important. Setting the \(\alpha\) is difficult knowing that the upper limit, target group, is changing and increasing and the Bandai has no access to weekly point of sales data. They initially fixed the diffusion speed to \(\alpha = 0.000000015\). The other important
8.2. THE TAMAGOTCHITM SUPPLY CHAIN NETWORK

A factor to be considered at market level is the phantom demand. Due to shortage of products, customers generate the phantom demand and some customers drop their plans to buy the product. In this work phantom demand is considered as 20% and dropout of customers as 10%. The dynamics associated with these two are adopted from the work of Higuchi [113].

At the retail level, we assume that the demands would be reviewed every week and forecasting of demands is performed by the suitable forecasting method. Generally qualitative forecasting methods such as subjective curve fitting techniques are used in forecasting for the new products as we don’t have the historical point of sales data. Experts with knowledge on the domain play a big role in these forecasts. However, in case of TamagotchiTM experts have no idea as it is the first of its kind in virtual pets. As there is no analogy existing with this product to other toy products, it is also difficult for the experts to provide accurate forecasts. Gopal [114] noted that the forecasts also depend on the ability of the supply chain apart from the customer demand. Bandai didn’t build the information systems of today in 1996 to apply the robust quantitative forecasting techniques. They could only consider the simple quantitative forecasting methods such as moving average method or exponential smoothing approach. Exponential smoothing can place greater emphasis on more recent data than does the moving average method [114], so in this work we choose exponential smoothing techniques as the forecasting method at retailers and manufacturer level.

At the manufacturer level, the production volume decisions are made once every week. The manufacturer needs time to increase the manufacturing capacity as it involves decision making at different stages and the resource. This delay to increase the manufacturing capacity is fixed as 3 weeks. The initial manufacturing capacity is fixed at 37,500 units. We used the system dynamics software, Vensim [115], as a tool to build the supply chain model. Vensim is particularly chosen as it is a simplified tool to build and analyze the models without any restrictions and casual tracing helps in identifying the flow of the effects. The main variables involved in our model and their dynamics are described Appendix A.

The stock and flow diagram [115] of the tamagotchiTM supply chain based on the relations shown in Appendix A is modeled in Vensim as shown in figure 8.5. In this thesis the main emphasis is placed in analyzing the dynamics of the diffusion speed and the repeat sales ratio assumptions on the performance of the SCN. These two assumptions are the key in designing the SCN and making decisions that have effect over long term on the manufacturer.
As Bandai spokeswoman Tomio Motofu said, Bandai increased the manufacturing capacity in order to meet with the increase in the demand [112]. As we have seen in section 8.2, the production realization from the increased manufacturing come after the 3 weeks. The manufacturing machines have a life of 3 years under normal use [113]. The depreciation of the machines is directly proportional to the usage of the machines. Apart from the machinery costs, the total costs involved in increasing the manufacturing capacity include personal recruitment and investments on land. The labor laws in terminating the employees is also an important issue in calculating the total costs. The dynamics involved with the decision of increasing the manufacturing capacity with respective to total demand is shown in figure 8.6. Figure 8.6 shows the sharp decline in
8.3. NONLINEAR DYNAMICS IN TAMAGOTCHI™ SUPPLY CHAIN NETWORK

the total demand after the considerable increase in the manufacturing capacity. As we discussed in section 8.2, this is also due to the reduction in the phantom demand and repeat sales requests from new customers knowing there are enough products in the market. Bandai increased their manufacturing facility just before the start of the decline in total demand due to overestimation and the lag between the identifying peak demand enhanced by phantom demand.

![Figure 8.6: Dynamics between total demand and manufacturing capacity in Tamagotchi™ SCN](image)

The increased capacity of the manufacturing facility to produce 2-3 million units per month from 100 thousand units per month resulted in a lot of obsolete inventory at the manufacturer as shown in figure 8.7. Due to lack of efficient information systems, and tools in aiding the investment policy the manufacturer bore the costs associated with the obsolete inventory. As we can see from figure 8.7, the retailers inventory is reduced after sometime but the manufacturers inventory is still the same. This phenomena shows the bullwhip effect at manufacturer. The retailer with the availability of immediate information from the market, reduces the orders or cancels the orders, but manufacturer with increased capacity is forced to produce the units as he saves only material cost of the product. This shift in inventories between the retailer and manufacturer is also due to the information lag between both of them.

![Figure 8.7: Dynamics between total demand and manufacturing capacity in Tamagotchi™ SCN](image)

From these two figures 8.6 - 8.7, we see that incase of the product with short life cycle, it is important to plan and analyze all the potential scenarios. The demand in the case of short product life cycle products grows fast. The manufacturers face more risk of phantom demand
created due to shortages and the risk of reduction in the number of potential customers. To analyze the possible scenarios the present solutions provided by IT are not sufficient. This inability may lead to a state where manufacturers have huge inventories but drop in sales. In this thesis, we propose the bifurcation analysis of the SCN as a method to analyze the possible scenarios for the short life cycle products.

8.4 Bifurcation analysis

It is usually impossible to assign a definitive, once-and-for-all valid number to most parameters occurring in the complex dynamical system such as SCN. For new products with short life cycle it is more difficult to assign the values as there is no previous historical knowledge base or experienced people. Bifurcation analysis as explained in section 5.4 provides the possible evolution of the system in long term (for example regular state, saturation state and chaotic state) as a function of the parameter of the system.
8.4. BIFURCATION ANALYSIS

8.4.1 Diffusion speed

Higuchi performed the analysis of the dynamics in the SCN by considering the different diffusion speed. He performed the analysis with three different values, 0.000000001, 0.000000015 and 0.000000020 [113]. He analyzed how the change in the diffusion parameters effected the total demand and manufacturing capacity related decisions. However, it is usually impossible to assign a definitive, once-and-for-all valid number to most parameters occurring in the complex dynamical system such as SCN. Also the theory from the nonlinear dynamic systems says that a very small variation in the parameter can lead the system to unstable state [26]. As Higuchi rightly pointed out the diffusion speed is an important parameter playing a key role in many managerial decision such as increasing manufacturing facility and inventory holding decisions [113]. By considering this in this work we performed bifurcation analysis on the diffusion speed in the parameter space between $0.1 \times 10^{-8}$ and $0.9 \times 10^{-8}$. The bifurcation analysis for the diffusion speed parameter space with respective to the inventory at manufacturer is shown in figure 8.8. The bifurcation analysis shows the islands of safety or islands of steady state of

![Bifurcation plot](image_url)

Figure 8.8: Bifurcation plot showing the sensitivity of the supply chain to the change of diffusion speed
the system in intervals between $0.38 \times 10^{-8}$ to $0.48 \times 10^{-8}$, $0.55 \times 10^{-8}$ to $0.65 \times 10^{-8}$ and between $0.75 \times 10^{-8}$ to $0.95 \times 10^{-8}$. Interesting the bifurcation diagram reveals that the system is stable in some intervals and chaotic in some intervals. This is an important feature of the bifurcation analysis to analyze the system over the complete parameter space instead of the hand picked values.

### 8.4.2 Rate of repeat

The second important parameter we considered in this thesis is the rate of repeat purchase. Even though the manufacturer did not get the full demand from the repeat purchase by the customers, it is included in the total demand generated by the customer. The customers for the repeat purchase look to alternate channels such as ebay than to regular retailers. By considering this we have performed bifurcation analysis of the repeat rate from 0.2 to 1. The impact of repeat rate on the inventory at manufacturer is shown in figure 8.9. The repeat rate below 0.6 has stable impact on the inventory at the manufacturer. Even though there are islands of stability between 0.65 to .7 it is advisable to assume that the repeat

![Figure 8.9: Bifurcation plot showing the sensitivity of the supply chain to the change of repeat purchase rate](image)

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rate is below 0.6 as the manufacturer is not enjoying the benefit of high repeat purchase rate fully.

8.5 Summary

The short life cycle products’ supply chain poses many challenges. These challenges are explained mainly from the perspective of external unchastities which are further divided into many categories. The two key categories that are specifically important in the context of the short life cycle products’ supply chain are namely design and volume. The generation of uncertainty due to the design relates to disturbances caused in the production flow by changes in the product design. This kind of uncertainty is more when the manufactured products go through numerous design changes within their life cycle. The volume of the production is the other key source for the generation of uncertainty in the short life cycle products’ demand prediction. During the introduction phase of the life cycle, little volume and variety is produced by manufacturers. The volume sizes increases in the growth phase. The type of SCN differs for each product type and is unique. The selection criteria for the case study and the background information required to understand the case study is described in detail in chapter 8. Thamagotchi\textsuperscript{TM} SCN is considered as a case study to illustrate the importance of analyzing the supply chains with theory and tools from nonlinear dynamic systems. The case study supply chain information is explained in detail from the state-of-the-art. The system dynamics modeling approach is explained in detail in this chapter. It is usually impossible to assign a definitive, once-and-for-all valid number to most parameters occurring in the complex dynamical system such as SCN. For new products with short life cycle it is more difficult to assign the values as there is no previous historical knowledge base or experienced people. Analysis of the nonlinear dynamics, with bifurcation theory, in Thamagotchi\textsuperscript{TM} SCN is presented and the importance of this tool is commented in detail in this chapter. Bifurcation analysis is performed on Thamagotchi\textsuperscript{TM} SCN for the two important parameters diffusion speed and rate of repeat purchase. These bifurcation diagrams give the "islands of stability” and help manufacturers reduce the risks.
Chapter 9

Nonlinear dynamics in lean supply chains

9.1 Introduction to lean concepts

Lean concepts can be traced to the evolution of the Toyota production system in the decades following World War II [116]. Even though lean concepts are not popular in the majority of the western world until the 1990s, basic lean principles focusing on manufacturing (better known as kanban, lean manufacturing, just-in-time production) had already evolved over many decades since the late 1940s. Two principle publications were instrumental in introducing the lean concepts and popularizing the benefits of lean concepts in supply chain. The first was "The Machine that Changed the World" by Womack et al. (1990) [116], which mainly summarized the results of the International Motor Vehicle Program at MIT during the previous five-year period. Womack introduced the evolution of lean production from craft production and mass production. Lean production provides the benefits of both the craft production and the mass production by providing the variety, the customer demands at much lesser price than the craft production by utilizing the specialized machines envisaged in mass production principles. The two key important aspects that enable the lean production are:

- Employing teams of multi skilled workers at all levels of the enterprise.
- Using highly flexible, increasingly automated machines to produce volumes of products in enormous variety.

Some of the key advantages observed by Womack [116] in implementing lean production are:

- Half the hours of engineering effort.
• Half the product development time.
• Half the investment in machinery, tools and equipment.
• Half the hours of human effort in the factory.
• Half the defects in the finished product.
• Half the factory space for the same output.
• A tenth or less of in-process inventories.
• 99.9% Customer schedule attainment.
• Functioning supplier partnership.
• Strong production control.

The second important publication, also from Womack, was *Lean Thinking* (1996)[35]. Womack reformulated and streamlined the core lean concepts that had evolved earlier into a structured change process to enable their effective implementation in an on-going mass production environment.

### 9.2 Importance of lean thinking

Lean represents a new and fundamentally different way of thinking about modern industrial enterprises. Lean thinking was defined by Womack and Jones to encompass four major steps:

1. Specify value.
2. Identify the value stream.
3. Make the value-creating steps for specific products flow continuously.
4. Let the customers pull value from the enterprise, and pursue perfection.

In order to understand the lean thinking better the four major steps are summarized in detail below:

• Value — Value is defined as something that the end customer is willing to pay for the product.
9.2. IMPORTANCE OF LEAN THINKING

- **Value Stream** — Mapping the value stream for each product provides a basis for performing an in-depth analysis of each of the individual actions in the value stream. Each action is classified into one of the following categories: (a) it unambiguously creates value, (b) it creates no value but is unavoidable given the current capabilities within the company, and (c) it creates no value and can be eliminated immediately [35].

- **Flow** — Once the wasteful actions along the value stream have been eliminated to the maximum extent possible, the next step is to make the remaining, value-creating steps “flow”. The major objective is transition from the batch-and-queue procedures from the mass production to small-lot production, ideal is batch sizes of a single unit, for the value-creating steps.

- **Pull** — The customer “pulls” the product from the enterprise rather than the enterprise pushing the product onto the customer. This “pulling” action cascades upstream, all the way to the supplier network.

“Lean thinking is the dynamic, knowledge-driven and customer-focused process by which all people in a defined enterprise continuously eliminate waste with the goal of creating value. Value delivery to the end customer is the main idea behind lean thinking, however the value creation is a central concept in lean thinking, to build robust, adaptive, flexible and responsive enterprises. In fact, value delivery may well miss the mark if it is not preceded by two essential, prerequisite, steps:

- **Value identification**, which involves identifying the stakeholders and their value needs or requirements.

- **Value proposition**, which defines the terms under which value exchanges among the stakeholders can take place.

In reality, value identification, value proposition and value delivery phases of the value creation process are closely coupled and highly interactive. Also, the idea of stakeholders is more broadly inclusive, even though focusing on the customer remains a critical feature of lean thinking. Lean Thinking, driven by the value creation framework just outlined, rests upon a set of mutually-reinforcing tenets or organizing principles [116, 35].
CHAPTER 9. NONLINEAR DYNAMICS IN LEAN SUPPLY CHAINS

9.3 Definition of lean supply chain network

A lean supply chain network is one that produces what is needed, how much is needed, when it is needed and where it is needed. Lean supply chain is not an extension of the traditional supply chain system; it is conceptually different from the traditional supply chain. Modern supply chain networks are undergoing a transition with mass customization, make-to-order, build-to-order and configure-to-order type of order policies, together with shrinking profit margins. Most of these modern supply chain processes are still fraught with unnecessary costs, excessive delays, and loads of inventory, which oppresses the response to customer needs. Without good SCN management, lean manufacturing alone does not work to its full potential. Contrarily the SCN will be very lethargic and unsusceptible unless top quality products are produced by the manufacturing part of the chain in response to customer demand at a minimal cost. The traditional pricing of products with cost plus approach is fast moving towards the cost reduction approach as shown in figure 9.1. This new necessity of cost reduction to remain in the competitive global market is forcing the organizations in reducing the wastes involved in the SCN by looking towards the lean concepts and to apply lean thinking to their SCN.

![Figure 9.1: Changing paradigm of pricing of products](image-url)

In most organizations, business processes are not properly linked and supply chains are disconnected from suppliers or internal/external customer needs. More over to deliver the value to the end customer organizations need to understand some basic concepts found in all Lean
Supply Chains. These are defined as:

- Product Flow - The value added movement of goods from the supplier’s raw material to delivery of final product to the customers.
- Customer Demand/requirement - The actual needs and demands of the customers.
- Information Flow - The flow of relevant data that supports the flow of product and service.
- Customer/Supplier Linkages - Key interfaces between customers and suppliers, which ultimately have an impact on movement of goods and services.

A lean supply chain integrates all four perspectives from internal order fulfillment to external work performed by suppliers, third party contractors and distribution networks. These activities make out what work needs to be performed, how it needs to be advanced, who are going to do it, and the priority in which the work is processed and completed. Internal logistics concentrates on relationship linking procurement, production, and delivery of goods in a seamless process. External logistics links the operations with service providers, suppliers, contractors and customers. Lean process is eliminating the wastes through out the supply chain network and yet guaranteeing the total quality for the product by providing exactly what the customer needs, when needed, in precisely the right quantity. The lean process is often misunderstood with the concept of layoffs, making people work fast, reducing inventories to very low levels in terms of cost cutting measures, which can only lead to a state of too lean. Implementing lean principles in the SCN involve sharing the best practices between enterprises, setting the benchmarks for the processes individually and for the supply chain as whole. The benchmark policies are made up to date by communicating with the other industrial competitors as shown in figure 9.2. The key supply chain lean processes are described in detail in subsequent sections.

9.3.1 Lean procurement

The procurement is a transactional process that affects many different departments within organization and SCN. End users need supplies, managers approve requests, buyers get the pricing and place orders, staff creates purchase orders and enters data into the computer, filing occurs, phone calls, the receiver unloads the orders and puts it away, he/she reconciles the receiving, the invoice goes to accounts payable to be paid [116].

Lean Procurement is a more efficient method to procure the multitude supplies that are used at your on a repetitive basis. Lean Procurement utilizes the principals of Lean Manufacturing to identify, recommend and implement a streamlined process. A main ingredient of Lean Procurement is utilizing e-commerce as a means to place orders, get confirmations, and obtain usage
reports for controls and invoicing, preferably one summary invoice at the end of a month. The benefits of Lean Procurement process can be as follows [117, 116].

- Remove the obstacles of the free flow of information to your supply chain.
- Transiting the supply chain from ”push” to ”pull”.
- Speed up the ordering process, the receiving process and invoice reconciliation process.
- Create real-time visibility into inventory in motion.

### 9.3.2 Lean manufacturing

Lean Manufacturing is in direct opposition with traditional manufacturing approaches characterized by use of batch-and-queue methods, high capacity utilization, and high inventory. Lean Manufacturing is an operational strategy oriented toward achieving the shortest possible cycle time by eliminating waste [11, 116].
9.3. DEFINITION OF LEAN SUPPLY CHAIN NETWORK

Lean Manufacturing is derived from the Toyota Production System (TPS) and its key thrust is to increase the value-added work by eliminating waste and reducing incidental work. Lean Manufacturing technique often decreases the time between a customer order and shipment, and it is designed to improve profit, customer satisfaction, throughput time, employee morale. The benefits generally are lower costs, higher quality, and shorter lead times. A lead time is a time period between the initiation of any process of production and the completion of that process. In any organizations, lead time reduction is an important part of lean manufacturing. The benefits of Lean Manufacturing can be as follows [11, 116].

- Increase inventory turns.
- Reduced work in progress inventories.
- Fewer redundant and valueless activities.
- Production possibility of variety of products.

9.3.3 Lean distribution

Lean distribution breaks the forecast accuracy barrier to improve customer service and profit with flawless execution of simplified operating processes. The efficiency of the supply chain network highly depends on the efficiency and agility of the distribution network. **Consolidation of shipments:** Consolidation of shipments is a great way to reduce moving costs considerably.

![Figure 9.3: Objectives of lean distribution in balancing the relationship between the supplier and customer [11]](image-url)
By shipment consolidation, goods are considered along with other consignments that are also being shipped in the same sector. Consolidation of shipments eliminate the wastage associated with the frequent empty or less than truck load shipments but they also restrict the flexibility [118, 35]. Shipment consolidation may also restrict the end value of the product as customer demands/requirements are hard to meet.

Core carrier programs: Core Carrier Programs have become an industry standard practice that allows companies to manage the supply chain at the earliest stage in the process. Strengthening relationships with "core" carriers brings many benefits in terms of cost and service including addressing special needs such as drop trailers, special loading and unloading instructions and priority unloading not to mention favorable cost and payment terms. Many shippers hired their opinion that the the savings from these programs deteriorate over time, while others fail to realize any financial benefit at all. Core carrier programs demand a long term approach from both the shipper and carrier which is again heavily depends on the forecasting accuracy [118, 35].

Cross docking: Cross docking is a practice of unloading materials from an incoming semi-trailer truck or rail car and loading these materials in outbound or rail cars, with little or no storage in between. Cross docking can be done to change type of conveyance, or to sort materials intended for different destinations, or to combine material from different origins [118, 35]. In the less than truck load trucking practice, which is general case of lean supply chain networks, cross-docking is done by moving cargo from one transport vehicle directly into another, with minimal or no warehousing. In retail practice, cross-docking operations may utilize staging areas where inbound materials are sorted, consolidated, and stored until the outbound shipment is complete and ready to ship.

Information technology: As information technology (IT) developed over the past decade, applications in distribution have multiplied. The IT provided the information visibility, availability for the optimization and planning modules. The popular solution in this regard is the transportation management system (TMS) [119]. TMS belong to a sub-group called supply chain execution (SCE). TMS usually "sits" between an ERP or legacy order processing and warehouse/distribution module.

TMS manage three key processes of transportation management [29, 120]:

- Planning and Decision: TMS defines most efficient transport schemes according to give parameters, which have a lower or higher importance according to the user policy: transport
9.3. DEFINITION OF LEAN SUPPLY CHAIN NETWORK

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<thead>
<tr>
<th>Paradigm</th>
<th>Traditional Approach</th>
<th>Lean Distribution</th>
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<tbody>
<tr>
<td>Customer service</td>
<td>Collaborate to forecast, then ship to film orders/releases</td>
<td>Manage flow as customer consumes, “owns” replenishment</td>
</tr>
<tr>
<td>Forecasts</td>
<td>Are accurate enough, but should strive to make more accurate</td>
<td>Limited accuracy, use for longer-term and aggregate planning only</td>
</tr>
<tr>
<td>Inventory</td>
<td>Is an asset and should be close to the customer to meet lead time demands</td>
<td>Consolidate at the source and redirect flow quickly for changing replenishment needs</td>
</tr>
<tr>
<td>Variability</td>
<td>Not explicitly used in planning, but measured in operations if Lean and Six Sigma are embraced</td>
<td>Operational, customer demand, and supply chain variability factors used in lean processes</td>
</tr>
<tr>
<td>Transportation</td>
<td>Changes with forecasts and orders; seek to reduce</td>
<td>Replenishment cycle driven; stabilize lanes to reduce</td>
</tr>
<tr>
<td>Optimization</td>
<td>Reduce each component of cost while filling forecasted demand</td>
<td>Stream line distribution total cost to replenish actual demand</td>
</tr>
<tr>
<td>Assumptions</td>
<td>Forecasts are sufficiently accurate and stable for planning</td>
<td>Pull reduces variation and improves service</td>
</tr>
<tr>
<td></td>
<td>All cost reductions add to net profit</td>
<td>Only total cost reduction adds to profit</td>
</tr>
<tr>
<td></td>
<td>Inventory costs less than labor</td>
<td>Inventory, handling, and storage costs are understated</td>
</tr>
</tbody>
</table>

Table 9.1: Comparison of paradigm changes between traditional and lean distribution approaches [13]

cost, shorter lead-time, fewer stops possible to insure quality flows regrouping coefficient.

- Transport follow-up: TMS will allow following any physical or administrative operation regarding transportation: traceability of transport event (shipping from A, arrival at B, customer clearance...), editing of reception, custom clearance, invoicing and booking documents, sending of transport alerts (delay, accident and non-forecast stops).

- Measurement: TMS have or need to have a Logistics key performance indicator (KPI) reporting function for transport.

A more complete comparison of the traditional approaches and lean approaches is provided in table 9.1, which tackles each of the main paradigms related to managing distribution.

9.3.4 Dynamics in lean supply chains

Customer order forecasts are the starting point for nearly all business, operational, and financial planning. To better manage lean SCN, besides forecasting customer order, customer requirements and needs also need to be forecasted with highest accuracy. Based on these fore-
casts, commitment to suppliers, setting / adjusting production and operational levels, making distribution arrangements and many other decisions are made in the quest for eliminating resource waste at different stages. These many decisions may require review, and change as forecasts are updated and revised, an almost daily process to keep pace with the market. This dynamic nature demands more accurate tools to understand the effect of the forecast errors, to review and to regenerate the plans without effecting the value of the product.

Distribution tends to be a major affected supply chain process with the issues and inaccuracy of forecast errors and market fluctuations, because excess or inadequate inventory is highly visible in a distribution center. Especially in lean supply chains where customer has the highest flexibility in his ordering and placing the requirement based on the current market-place trends and competition the fluctuations are more. There are immediate impacts to service in distribution for inventory imbalances, many of which are due to this market-place dynamics. So the success of distribution highly depends on the ability to generate / re-generate the optimized plans dynamically with respective to the dynamics of the customer orders and requirements.

9.4 Importance of real time resource allocation in lean distribution

Resource allocation is one of the important tasks/functions in managing lean supply chain network many processes such as transportation, production, and order fulfillment [120]. The main aim of resource allocation is to utilize the available resources optimally and minimize the cost or maximize the profit by eliminating the "waste". Distribution in lean environment is dynamic and the scheduling/re-scheduling is expected in short notice. The transportation scheduling problem has been well studied in the past six decades where many researchers have developed diverse methods for solving the transportation scheduling problem. The standard transportation problem was first presented by Hitchcock in 1940’s [121], which can be expressed as minimization of transportation cost for carrying commodities from \( m \) sources to \( n \) destinations while operating within supply and demand constraints. This problem can be modeled as a linear programming problem. The lean transportation problems are complex and dynamic in nature and do therefore need or call for appropriate techniques to solve them in real-time. Transportation problems generally involve some form of scheduling for vehicles, equipments and goods. From that perspective transportation problems can be seen as planning or scheduling problems. The planning is done for a specific time horizon and based on that time window transportation planning problems are generally classified as strategic, tactical and
The availability of information is crucial for planning a schedule. This information is available at different stages for different problems such as static problems, stochastic problems and dynamic problems. Information in static problem is available well in advance and in stochastic problems the availability of information is at the time of scheduling. In the case of the dynamic problems no information is available prior to the situation but the information is gradually revealed over time. Some times it needs re-planning in stochastic problems when some random events occur. In case of the dynamic problems planning must be done in real time. In order to perform the real time planning, data processing must be fast and accurate. Most of the problems in transportation are dynamic in nature because of the unpredicted situations such as traffic congestion, random truck or vehicle failure etc. The schedule in dynamic problems must be released before the time window reaches the deadline.

The key objectives of this thesis are mainly threefold: (1) to show the potential of the concepts involving analog computing for designing highly efficient algorithms for transportation scheduling in lean environment; (2) to demonstrate the efficiency of a novel analog computing scheme based on cellular neural networks while used for scheduling and optimization; and (3) For illustration purposes, to demonstrate the ultra-fast (respectively real-time) solution of some selected dynamic transportation problems involving re-scheduling.

9.4.1 Traditional approaches

Transportation scheduling problems are studied and well developed from the past six decades. The major technologies that contributed to solve these problems vary from crude rules to most sophisticated searching algorithms and artificial intelligence (AI). Some of the approaches to solve the scheduling issues proposed in the literature are: heuristic rules, mathematical programming techniques, neighborhood search methods and AI techniques. These approaches have their own limitations as heuristics provide no guaranty for optimization, mathematical methods cannot solve NP-hard problems [122, 123, 124, 125], neighborhood techniques [126, 127, 128, 129, 130] and AI based techniques get trapped in local optima [131].

These scheduling techniques also have a common limitation in terms of computation time. The computation time is an important criterion in real time scenario where schedule is needed in very short time. The real time scenario is dynamic in nature i.e. it always needs rescheduling when new tasks are included at run-time or when some existing tasks are excluded. The scheduling techniques so far evolved take more computation time which is almost a limitation.
when dealing with real time scenarios.

As an attempt to minimize the computation time we introduce an analog computing method for solving scheduling problems. The main idea is to solve a scheduling problem by using Cellular Neural Networks (CNN). CNN is an analog computing paradigm which handles differential equations effectively. In the following sections we introduce the basics of CNN processor and the method of solving differential equations using CNN processor.

**Lagrangian relaxation**

In the early 1970’s, the optimization problem is represented with a simple objective function subjected to complex constraints. These complex constraints make the problem hard. In order to solve these hard problems the constraints are relaxed using some relaxation methods. There are so many relaxation methods in use to make a hard problem simple.

Lagrangian relaxation is a relaxation technique widely used for the linear programming problem. Some of the characteristics of the Lagrangian relaxation method are described in [132, 133].

Let \( P \) be an optimization problem of the form

\[
\text{min} \{ f(x) \mid Ax \leq b, Cx \leq d, x \in X \} \tag{9.1}
\]

where \( X \) may contain non-negative integers or any other set which restricts the solution set of the optimization problem. The constraint \( Ax \leq b \) is assumed to be a complicated one. Then the Lagrangian relaxation of \( P \) can be given by relaxing the complicated constraint keeping the other constraints unmoved. The Lagrangian relaxation of \( P \) relative to the complicating constraint \( Ax \leq b \), with non-negative Lagrangian multiplier \( \lambda \), is defined as follows

\[
\text{min} \{ f(x) + \lambda(Ax - b) \mid Cx \leq d, x \in X \} \tag{9.2}
\]

Here the complicated constraint \( Ax \leq b \) is added to the objective function with weight \( \lambda \) and it is released from the constraint list. This is known as dualized problem. The problem is relaxation of \( P \), since for any \( x \) feasible for \( P \) and for \( \lambda \geq 0 \), \( f(x) + \lambda(Ax \leq b) \) is less than or equal to \( f(x) \). In the similar way we can relax the other constraints and the optimization equations is expressed as follows:

\[
\text{min} \{ f(x) + \lambda_1(Ax - b) + \lambda_2(Cx - d) \mid x \in X \} \tag{9.3}
\]
9.5. CELLULAR NEURAL NETWORKS BASED REAL TIME SCHEDULING

In order to minimize the given function $f(x)$, we calculate suitable values for the lagrangian multipliers.

9.5 Cellular neural networks based real time scheduling

The main idea behind lean scheduling is to increase customer satisfaction by providing goods at right time while utilizing the resources properly. The scheduling problems are difficult to solve because of their NP-hard nature. To solve the hard problems so many approaches are specified which are time consuming [122, 123, 124, 125]. This limitation of solving hard problems in reasonable time by existing methods motivated to propose a new approach. One such techniques which handles hard problems effectively is cellular neural networks(CNN). CNN is an analog simulation technique with parallel processing. CNN processors have been used for researching non-equilibrium systems, constructing non-linear systems, dynamic systems, studying emergent chaotic dynamics, generating chaotic signals. The major advantages of CNN are:

• Effective to analyze ordinary differential and partial differential equations.
• Much faster when compared to other regularly used methods.
• Can be implemented in hardware which needs very less space and energy.

9.5.1 Introduction to cellular neural networks

The concept of CNN, also called Cellular Non-linear Network was introduced in 1988 by Leon O.Chua and Lin Yang. The original idea was to use an array of simple, non-linearly coupled dynamic circuits to parallely process large amounts of data in real time [134, 135, 136]. It is a large array of interconnected nonlinear dynamic systems called cells. It can be identified as the combination of cellular automata [137] and neural networks [138].

The CNN processor with a self feedback is modeled by Eq. 9.4, with $x_i$, $y_{ij}$ and $u_i$ as state, output and input variables respectively.

\[ \dot{x}_i = -x_i + \sum_{(j) \in N_r(i)} (\hat{A}_{ij}x_j + A_{ij}y_{ij} + B_{ij}u_j) + I \]

\[ y_{ij} = \frac{1}{2}(|x_{ij} + 1| - |x_{ij} - 1|) \]

The coefficients $\hat{A}_{ij}$, $A_{ij}$ and $B_{ij}$ in Eq. 9.4 are the self feedback template, feedback template
The schematic model of a CNN cell with a self feedback is shown in figure 9.4. CNNs are particularly interesting because of the programmable nature i.e. changeable templates. The major applications of CNN are image processing, solving non-linear dynamics which can be modeled by ordinary and partial differential equations.

9.5.2 Solving optimization problems with cellular neural networks

In order to solve a scheduling problem using CNN processor, we model the scheduling problem by a set of differential equations. These differential equations are compared with the CNN state equation Eq. 9.4a to obtain the appropriate template values for \( \hat{A} \), \( A \) and \( B \). The transformation of scheduling problem into a set of differential equations is achieved by using Lagrangian relaxation method.

In our approach, we consider the lagrangian multipliers as additional variables and we find the optimum values for these multipliers. In order to find the optimum values for all the variables using CNN processor, we find the differential equations for those variables. The differential equations for each variable are obtained by finding the partial derivative of the lagrangian dual with respect to the corresponding variable. In the next section, the process of solving a scheduling problem using the CNN processor is explained in detail.
9.5.3 Scheduling a transportation problem with cellular neural networks

It is well known from recent literature that CNN processors are capable of solving ordinary and partial differential equations very efficiently [139]. Thus, the next logical step for solving a scheduling problem using CNN processors is to transform the problem into a set of differential equations. For this process a scheduling problem is modeled as a linear programming problem as shown:

Assume that there are $I$ ports, $P_1, ..., P_I$, that supply a certain commodity, and there are $J$ markets, $M_1, ..., M_J$, to which this commodity must be shipped. Port $P_i$ possesses an amount $s_i$ of the commodity ($i = 1, 2, ..., I$, and market $M_j$ must receive the amount $r_j$ of the commodity ($j = 1, ..., J$). Let $b_{ij}$ be the cost of transporting one unit of the commodity from port $P_i$ to market $M_j$. The problem is to meet the market requirements at minimum transportation cost.

The variables in the model are described below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Number of ports</td>
</tr>
<tr>
<td>$J$</td>
<td>Number of markets</td>
</tr>
<tr>
<td>$P_i$</td>
<td>$i$th Port</td>
</tr>
<tr>
<td>$M_j$</td>
<td>$j$th Market</td>
</tr>
<tr>
<td>$s_i$</td>
<td>Commodity supplied from port $P_i$</td>
</tr>
<tr>
<td>$r_j$</td>
<td>Commodity required at market $M_j$</td>
</tr>
<tr>
<td>$y_{ij}$</td>
<td>The quantity of the commodity shipped from port $P_i$ to market $M_j$</td>
</tr>
<tr>
<td>$b_{ij}$</td>
<td>Cost of transporting one unit of the commodity from port $P_i$ to market $M_j$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Lagrangian multiplier</td>
</tr>
</tbody>
</table>

Let $y_{ij}$ be the quantity of the commodity shipped from port $P_i$ to market $M_j$. The total transportation cost is given as

$$\sum_{i=1}^{I} \sum_{j=1}^{J} y_{ij} b_{ij}.$$  \hspace{1cm} (9.5)

The amount of commodity shipped from port $P_i$ is $\sum_{j=1}^{J} y_{ij}$, and the amount of commodity available at port $P_i$ is $s_i$

We must have

$$\sum_{j=1}^{J} y_{ij} \leq s_i \text{ for } i = 1, ..., I$$  \hspace{1cm} (9.6)

The amount of commodity sent to market $M_j$ is $\sum_{j=1}^{I} y_{ij}$, and the amount required at $M_j$ market...
is \( r_j \), i.e.,

\[
\sum_{i=1}^{I} y_{ij} \leq r_i \text{ for } j = 1, \ldots, J \tag{9.7}
\]

We cannot send a negative amount from \( P_i \) to \( M_j \), So

\[
y_{ij} \geq 0 \text{ for } i = 1, \ldots, I \text{ and } j = 1, \ldots, J \tag{9.8}
\]

Our aim is to minimize Eq. 9.5, subjected to the constraints Eq. 9.6-9.8.

The method of solving an optimization problem by using the CNN processor is explained in figure 9.5 along with the theoretical description of the proposed approach. The main idea of this method is to transform a mathematical programming problem as a set of differential equations and solving those differential equations by using CNN processor. The process of transforming mathematical programming problem into a set of differential equations is performed in two steps:

1. First step is to transform the scheduling problem, modeled as a mathematical programming problem, into a single equation i.e., lagrangian dual:
   - This is performed by relaxing the constraints i.e., moving the constraints into the objective function by applying the Lagrangian relaxation method.
   Applying the lagrangian relaxation to the scheduling problem described in Eq. 9.5, subjected to constraints Eq. 9.6-9.8 the lagrangian dual for the problem is given as.

\[
L(\lambda) \equiv \min_{x} \left[ \sum_{i=1}^{I} \sum_{j=1}^{J} y_{ij} b_{ij} + \lambda_1 (\sum_{j=1}^{J} y_{ij} - s_i) + \lambda_2 (\sum_{i=1}^{I} y_{ij} - r_i) \right] \tag{9.9}
\]

2. The second step is transforming the obtained lagrangian dual into a set of differential equations and this can be done by
   - Considering the Partial derivative of the lagrangian dual equation, obtained by applying the Lagrangian relaxation method, with respect to each variable in the equation.
   - Then according to the neuron dynamics the signs are assigned to each variable (differential equation).
   - For decision variables the sign is negative and for Lagrangian multipliers the sign is positive.
   For the lagrangian dual in Eq. 9.9 the differential equations are
The differential equations obtained are solved by using CNN processors. The process of solving the differential equations by using CNN processor is performed as follows:

- The number of differential equations, to be solved, defines the maximum index of the CNN processor array.
- Then the differential equations are arranged in the similar form as CNN processor’s state equations. This will make the coefficients comparison of the variables, to the corresponding equation, simple.
- The coefficients of variables are compared, to that of the corresponding CNN processor state equation, to obtain the template values for each CNN processor.
- These template values are used to implement the CNN processor in Matlab/ Simulink.

The CNN processor is simulated on top of Matlab/Simulink till a stable optimum result, for each variable involved in the equations, is obtained. The time taken to get a stable optimum result is the computation time taken by CNN processor to generate the result.

**9.5.4 Case study for illustration**

The quadratic programming problem illustrated in figure 9.5 with the objective function is given in Eq. 9.13.

\[
\min c_1 x_1^2 + c_2 x_2^2
\]  

Subjected to the constraints in Eq.9.14 and Eq.9.15.

\[
a_1 x_1 + a_2 x_2 \geq b_1 \quad (9.14)
\]

\[
a_3 x_1 + a_4 x_2 \geq b_2 \quad (9.15)
\]

\[
x_1, x_2 \in R \quad (9.16)
\]
This problem is solved by using CNN processor for different \((a_1, a_2, a_3, a_4, b_1, b_2, c_1, c_2)\) values. A similar quadratic programming problem is solved by Luh et al., in [140] by using neural networks for the parameter values:

\[
\begin{bmatrix}
  a_1 & a_2 \\
  a_3 & a_4 \\
\end{bmatrix} = \begin{bmatrix}
  1 & -0.2 \\
  5 & 1 \\
\end{bmatrix}, \quad \begin{bmatrix}
  b_1 \\
  b_2 \\
\end{bmatrix} = \begin{bmatrix}
  48 \\
  250 \\
\end{bmatrix}, \quad \begin{bmatrix}
  c_1 \\
  c_2 \\
\end{bmatrix} = \begin{bmatrix}
  0.5 \\
  0.1 \\
\end{bmatrix}
\]

(9.17)

In order to demonstrate the superiority of CNN processor, we solved the problem with the same parameter set and the optimum results for \((x_1, x_2, \lambda_1, \lambda_2)\) are obtained as \((49, 5, 22, 5.9)\).
9.5. CELLULAR NEURAL NETWORKS BASED REAL TIME SCHEDULING

The trajectory of neurons was studied for each variable and the results were plotted by Luh et al, in [140]. Figure 9.6(a) shows the values of variables and the number of iterations taken by neural networks to obtain the optimum result. It is shown that neural networks take around 2000 iterations to obtain a stable optimum result for the variables.

The trajectory of neurons obtained from the CNN processor is plotted in figure 9.6(b). The CNN processor takes around 200 iterations to obtain stable optimum results. Considering the number of iterations in both the cases, it appears that the CNN processor takes less iterations for convergence (i.e. obtention of stable optimum result) when compared to neural networks. This results to an improvement of the computation time when using the CNN processor with a speedup of approximately 10. Therefore the CNN processors are very fast (less computation time) when compared to neural networks approach to solve optimization problems.

![Figure 9.6: (a) Trajectory of neurons for each variable in neural networks, (b) Trajectory of neurons for each variable in CNN processor](image)

Different types of scheduling problems have been solved using the novel CNN based computational approach proposed in this work, in order to evaluate the efficiency of the proposed method. The results obtained from the CNN based approach are compared with the results from traditional operations research based approaches. The proposed approach has been evaluated for a specific scheduling problem with the number of ports $P_i$ and markets $M_j$ varying from 3 to 10. The computation time to get an optimum plan is compared with that of quadprog and linprog functions (Matlab built-in function for solving optimization problems).

Computation time required for an optimal scheduling plan for the specific problem with the CNN based method and Matlab built-in function for solving optimization problems is plotted.
in figure 9.7. It shows that the computation time for CNN based solver increases linearly with the dimension of the problem, whereas for the Matlab in-built functions for solving optimization problems, the variation increases exponential. This shows the superiority of the CNN computing paradigm when compared to the classical approaches to solve scheduling problems.

### 9.5.5 Interpretation of results

The main contribution in this work is the development and validation of a consistent methodology for transforming and reformulating any scheduling and optimization problem into a set of differential equations. Then we propose a highly efficient analog computing concept/platform based on CNN processors for solving the (eventually nonlinear) differential equations obtained. Diverse scenarios have been considered in the experimental and validation parts of this work: (a) optimization problems; (b) extension to some scheduling problems. The results have been compared with other approaches and published works from literature. The comparison criteria have been the exactitude of the results and the computation speed. It could clearly been demonstrated that the CNN based analog computing approach for scheduling is ultra-fast and produce excellent results when compared to other traditional approaches. The scalability of the novel approach could also be confirmed, since its computational time increases only linearly with the problem size, whereas all traditional ones generally display an exponential increase of the computational time. In this work, the CNN processors (that are the heart of the analog computing platform) have been efficiently implemented in Simulink; however, a hardware implementation on DSP or FPGA is also thinkable and planned for the future.
9.6 Summary

The potentials of methods/tools from nonlinear dynamic systems theory in lean SCNs’ is described in this chapter. To better understand the definition of the lean SCN, the lean principles and the lean thinking are explained in detail before introducing the lean SCN. The lean concepts at various levels in the SCN explained in this chapter also gives detailed information on importance of lean supply chains. The nonlinear dynamics of the lean supply chain are discussed theoretically in detail. Resource allocation is one of the important tasks/functions in managing lean supply chain networks many processes such as transportation, production, and order fulfillment. The main aim of resource allocation is to utilize the available resources optimally and minimize the cost or maximize the profit by eliminating the "waste". Distribution in lean environment is dynamic and the scheduling/re-scheduling is expected in short notice. Importance of real time scheduling in lean supply chains is presented in conjunction with importance of dynamic resource allocation in lean SCN. Application of cellular neural networks based analog computing approach for dynamic resource allocation is described in detail in this chapter. An introduction to cellular neural networks is also presented to better understand its usefulness in application requiring high speed and complex computations.
CHAPTER 9. NONLINEAR DYNAMICS IN LEAN SUPPLY CHAINS
Chapter 10

Conclusion

10.1 Summary of the work

In this thesis, an extensive review of the literature on SCN management was conducted to understand the evolving trends of managing SCNs’ and the dynamics involved in managing the SCNs’. It is observed from the literature that the entities in the SCN are coupled, entities such as customer and retailer are loosely coupled (no fixed contracts or relationships) and entities such as manufacturer and distributor are tightly coupled (fixed contracts), and the effects of uncertainties propagate in both upstream and downstream. This makes the primary objective of the modeling approach to include the market induced uncertainties into the model, to quantify the effects of uncertainties and to mitigate the effects. State-of-the-art modeling methods in SCN have been studied by classifying them into modeling frameworks by standard organizations and simulation based modeling approaches that are developed to analyze and optimize SCN. Different modeling approaches are compared over a set of comparison criteria, such as ability to quantifying the dynamics, ability to deal with uncertainties, scalability and adaptability of the models, to list their merits and demerits. The traditional modeling approaches such as system theory, agent technology and petri-nets have limited ability to quantify the effects of uncertainties along the supply chain, both in upstream and downstream, and limited scope for the mitigating the effects of uncertainties. The mathematical programming based modeling approaches especially depend on the clear definition of variables. To some extent discrete event simulation modeling approach considers the dynamic nature of variables. The details of the finding are listed in chapter 4.

In this thesis, the "coupled oscillatory systems" based modeling method from the dynamic systems theory has been proposed for the investigation of the effects of external uncertainties.
A simple SCN consisting of manufacturer, distributor and retailer (three levels) is considered for modeling the generic SCN that can represent the broad SCN principles. Related mathematical models were derived by exploiting and considering important parameters that can effect the key performance indicators of the SCN. The dynamics of this SCN were modeled mathematically by the well-known Lorenz oscillator. From the theory of nonlinear dynamic systems it is proved that this model produces a wide variety of nonlinear features depending upon the parameters' values. This model is particularly of interest when dealing with the modeling of scenarios/phenomena which are very sensitive to initial conditions and to both internal and external uncertainties as well. An example SCN is defined by assigning the optimum values for the key parameters in the SCN model to consider it as a reference SCN model for the further analysis. The description of the model and the mathematical formulation of the model are described in chapter 6.

After the appropriate modeling of supply chain network through a coupled oscillator model, two important analysis methods from the nonlinear dynamic systems theory, namely "phase portrait analysis" and "bifurcation analysis" have been used to extensively analyze all possible behaviors of an abstract and theoretical reference SCN network subjected to external uncertainties. The form and structure of the phase portrait is used to reveal the information about the supply chain behavior for the chosen parameter values under the influence of the external fluctuations: in case of the form of a point, the system is in saturation; in case of a periodic orbit, the system is in a regular state; and in case of discernible patterns, the system is in a chaotic state. The example supply chain phase portrait reveals a periodic orbit indicating that the example system is in the regular state before subjecting to the external fluctuations. However, after subjecting to the external fluctuations, the form and structure of the phase portrait varied from a point to discernible patterns indicating that the supply chain is going through different states such as saturations and chaos. The other important method that gives more broader view on the stability of the system over a wide parameter range is bifurcation analysis. The bifurcation analysis is carried out for two important internal parameters (i.e., rate of customer demand satisfaction and rate of information distortion at distributor) to illustrate the effectiveness of the bifurcation analysis. It has been found through bifurcation diagrams that the effects due to fluctuations can lead to both chaotic and regular states of the supply chain and that these states alternate when monitoring the internal parameters of the supply chain. The bifurcation analysis in this work has been shown to be necessarily important as it could help the decision makers at various levels such as strategic, tactical and operational levels, to better understand the performance of the supply chain over a range of parameter space. Therefore, the new modeling approach paired with the two analysis instruments have demonstrated their capability to really assist in better simulat-
ing and understanding the highly complex nonlinear behavior of real supply chain networks. The details of these two methods and the significance of these methods is explained in chapter 6.

After this, the next challenge has been that of designing a control strategy to stabilize the supply chain network against external perturbations, especially when these could lead its behavior into a chaotic state. Concerning the stabilization of the supply chain network against external perturbations and/or uncertainties, a control strategy based on adjusting internal parameters of the supply chain network has been developed and demonstrated. A control scheme is conceptualized and designed based on "active control algorithm" to cancel or alleviate the effects due to the external fluctuations. It is also shown that this cancelation or alleviation leads to the achievement of synchronization which is characterized by the re-establishment of the reference example supply chain. The control process with active control algorithm is based on the variation of the internal parameters of the supply chain in well-specified ranges obtained thorough the bifurcation analysis. These are ranges within which the achievement of regular state is possible and the range is acceptable for all the stakeholders of the supply chain. The details of the active control algorithm and its suitability to SCNs is described in chapter 7.

The preciousness of this novel modeling, analysis and control approach based on nonlinear dynamics concepts is very appealing especially in a time of innovative products with short product life cycles. The short life cycle character of these products demands understanding of the affects of external fluctuations on the supply chain even before they start the production as the efficiency and performance of the supply chain is directly related to the success of the product. By considering this, in this dissertation as a case study virtual pet toy, Tamagotchi™, products’ supply chain is considered to illustrate the usefulness of performing bifurcation analysis on the supply chain. Tamagotchi™ was the first of the virtual pet games, introduced in 1996 by the Japanese toy manufacturer, Bandai Co. This case provides a good example illustrating the problems that can arise from the interactions between unpredictable demand, boom or bust, and capacity decisions in the short product life cycle setting. Even though the product Tamagotchi™ is a tremendous success in the market, the Bandai Co lost 16 billion Yen in 1998. Through bifurcation analysis we have showed how Bandai Co could have avoided the losses by analyzing the system over a set of parameter space. The details of the case study and the bifurcation analysis to the Tamagotchi™ SCN are explained in chapter 8.

The management of a special type of supply chain networks with strict/high performance requirements, called "lean supply chain networks", involves huge real time planning and re-planning processes especially due to the inherent dynamics of the real external environment.
Just-in-time manufacturing schemes are examples of some of the publicly well-known concepts related to such SCN concepts. Much of those (re-)planning processes are essentially a matter of resource (re-)allocation and scheduling. Such problem settings are generally computationally NP-hard. Thus, speeding-up the real-time planning processes in such "lean supply chain networks" is a key challenge. The nonlinear dynamics area has provided inspiration to developed a novel ultra-fast scheduling scheme involving an emulated "analogue computing" concept based on cellular neural networks. This new concept has proven to be scalable, as its computational complexity grows linearly against problem size. It therefore clearly outperforms competing traditional approaches, which have a rather, in the trends, exponentially growing complexity. The extremely high speeding-up of the scheduling processes does also enable a better reaction of the supply chain to fast internal or external changes and thereby significantly contribute to ensuring the stabilization of its overall performance and behavior.

Various modeling and simulation approaches are studied. The modeling and simulation approaches proposed in the literature have limited ability to quantify the effects of uncertainties along the supply chain, both in upstream and downstream, and limited scope for the synchronization and control of the effects of uncertainties. It is also observed from the literature that the entities in the SCN are coupled, entities such as customer and retailer are loosely coupled (no fixed contracts or relationships) and entities such as manufacturer and distributor are tightly coupled (fixed contracts), and the effects of uncertainties propagate in both directions. By considering this phenomenon, coupled oscillatory systems based modeling approach from the dynamic systems theory is proposed for the investigation of the effects of uncertainties generated by the dynamic and volatile global market-place on the stability of the supply chain.

A simple SCN consisting of manufacturer, distributor and retailer (three levels) is considered for modeling the generic SCN that can represent the broad SCN principles. The mathematical models were derived by exploiting and considering key parameters that can affect the key performance indicators of the SCN structure proposed in this work with the assumption that the demand information is transmitted within the levels of the SCN with a delay of one unit of time. The dynamics of this SCN were modeled mathematically by the well-known Lorenz oscillator. From the theory of nonlinear dynamic systems it is proved that this model produces a wide variety of nonlinear features depending upon the parameters’ values. This model is particularly of interest when dealing with the modeling of scenarios/phenomena which are very sensitive to initial conditions and to both internal and external uncertainties as well. An example SCN is defined by assigning the optimum values for the key parameters in the SCN model to consider it as a reference SCN model for the further analysis. The two important tools/methods from
the nonlinear dynamic systems theory namely phase portrait analysis and bifurcation analysis is introduced to analyze the dynamics of the reference supply chain model when subjected to uncertainties. The uncertainties are considered as periodic external fluctuations such as demand fluctuations, supply fluctuations, delivery fluctuations, and forecasting fluctuations. The effect of these fluctuations on the reference supply chain model is analyzed with both phase portraits and bifurcation diagrams.

The form and structure of the phase portrait is used to reveal the information about the supply chain behavior for the chosen parameter values under the influence of the external fluctuations: in case of the form of a point, the system is in saturation; in case of a periodic orbit, the system is in a regular state; and in case of discernible patterns, the system is in a chaotic state. The example supply chain phase portrait reveals a periodic orbit indicating that the example system is in the regular state before subjecting to the external fluctuations. However, after subjecting to the external fluctuations, the form and structure of the phase portrait varied from a point to discernible patterns indicating that the supply chain is going through different states such as saturations and chaos. These two modes show the states of this instability of the supply chain and the causes of this instability can be broadly referred as the dynamical and non-linear character of the motions (i.e. material/products flow and information exchanges) between different entities in supply chains because of the external fluctuations. A control and synchronization scheme is conceptualized and designed based on active control algorithm and exploited to cancel or alleviate the effects due to the external fluctuations. It is also shown that this cancelation or alleviation leads to the achievement of synchronization which is characterized by the re-establishment of the reference example supply chain. The control and synchronization process is based on the variation of the internal parameters of the supply chain in well-specified ranges. These are ranges within which the achievement of synchronization is possible and the range is acceptable for all the stakeholders of the supply chain.

In order to derive or determine these well-specified ranges for the key internal parameters which can be adjusted to achieve the synchronization, bringing back to reference example supply chain, without affecting any of the stakeholders bifurcation analysis of the supply chain model is proposed. Bifurcation diagrams are particularly chosen as they reveal the states (equilibrium/fixed points, periodic or chaotic states) of the system in a given parameter space. The bifurcation analysis is carried out for two important internal parameters (i.e., rate of customer demand satisfaction and rate of information distortion at distributor) to illustrate the effectiveness of the bifurcation analysis. It has been found through bifurcation diagrams that the effects due to fluctuations can lead to both chaotic and regular states of the supply chain.
chain and that these states alternate when monitoring the internal parameters of the supply chain. The bifurcation analysis in this work has been shown to be necessarily important as it could help the decision makers at various levels such as strategic, tactical and operational levels, to better understand the performance of the supply chain over a range of parameter space.

The importance of both these methods from nonlinear dynamic systems, phase portrait analysis and bifurcation analysis, to analyze and to aid in control and synchronization of the supply chain is very appealing in a time of innovative products with short product life cycles. The short life cycle character of these products demands understanding of the affects of external fluctuations on the supply chain even before they start the production as the efficiency and performance of the supply chain is directly related to the success of the product. By considering this, in this dissertation as a case study virtual pet toy, Tamagotchi™, products’ supply chain is considered to illustrate the usefulness of performing bifurcation analysis on the supply chain. Tamagotchi™ was the first of the virtual pet games, introduced in 1996 by the Japanese toy manufacturer, Bandai Co. This case provides a good example illustrating the problems that can arise from the interactions between unpredictable demand, boom or bust, and capacity decisions in the short product life cycle setting. Even though the product Tamagotchi™ is a tremendous success in the market, the Bandai Co lost 16 billion Yen in 1998. The system dynamics based model of the supply chain is adopted from the literature and the bifurcation analysis is carried out on the model to illustrate how they might have avoided these tremendously unfortunate effects.

The significance of theory from nonlinear dynamics in managing lean supply chains is also investigated to find which other tools/methodologies can be exploited to better implement lean thinking in supply chains. Managing lean supply chains demand dynamic resource allocation and scheduling at various stages, especially at distribution stage. Lean distribution approach increases flexibility and reduces the reliance on forecasts and optimized plans to achieve the optimum performance. But this demands ultra fast scheduling and re-generation of optimum plans. In this thesis, we propose a novel cellular neural network based analog computing paradigm to generate optimum plans with higher speeds. A theoretical overview is also presented on integrating the nonlinear dynamic systems tools/methodologies in a real time enterprize resource planning application’s supply chain modules.
10.2 Outlook

The integration of the modeling and simulation of SCN using the theory from the nonlinear dynamic systems with the present enterprise resource planning (ERP) solutions focusing on supply chain management is very interesting, even though it is out of scope of this thesis. This integration facilitates and extends the scope of the ERP solutions from the intra-organizational planning and optimization solutions to inter-organizational solutions. This type of integration also improves the modeling and simulation as we have access to the real-time data from the ERP. Integrated-ERP solutions provide the decision makers at various stages, especially at operational level, with more information as the ERP can give the real-time situation and the nonlinear dynamics based simulation can provide details on how the situation can evolve and what are the consequences and remedies. The nonlinear dynamics based simulation, analysis and control can be integrated as separate module like the supplementary modules such as the Inventory Collaboration Hub (ICH) and Extended Warehouse Management (EWM) modules in SAP supply chain management software suite.

Additionally, it is also important to take into consideration that the ERP solutions are not deployed by all the players in the SCN. Some small companies can’t afford to deploy ERP solutions but they have stand alone advanced planning systems in place. The proposed concepts can be further developed so that the outcome of these methods can be easily interfaced with the standalone applications such as Transportation Management Systems (TMS), Inventory Management Systems (IMS) and production scheduling systems.

It is also very interesting to develop the proposed methods on the hardware platforms (with novel analog computing paradigms) as it will increase the response time of the solutions. This is particularly an important issue in managing the lean supply chains. Lean SCNs demand scheduling/re-scheduling of plans and resource allocation at shorter notice. The cellular neural network based analog computing paradigm presented in this thesis to illustrate its efficiency in the filed of resource allocation can be further investigated to apply these methods for wide range of optimization issues involved in managing SCN.
Appendix A

Equations used in the Tamagotchi™ supply chain model

(01) A = 
3e-008 
Units: **undefined**

(02) Accumalated Order = INTEG ( 
Retail Order-Received, 
0) 
Units: **undefined** [0,?] 

(03) Alpha = 
0.2 
Units: **undefined**

(04) Back log = 
MAX(Total Demand-Periodical Sales, 0) 
Units: **undefined** [0,?] 

(05) Complete = 
DELAY FIXED(IF of UC, 3, 0) 
Units: **undefined**

(06) Diffusion Level = INTEG (}
APPENDIX A. EQUATIONS USED IN THE TAMAGOTCHI™ SUPPLY CHAIN MODEL

Periodical diffusion, 75000
Units: **undefined** [0,?] 

(07) Double=
IF THEN ELSE(Working Rate>1, 1, 0)
Units: **undefined**

(08) Expansion=
IF THEN ELSE(Upward=1, MAX((Expected Demand-Manufacturing Ability-Under Construction)/Investment Policy,0), 0)
Units: **undefined**

(09) Expected Demand= INTEG ( 
IF of ED-OF of ED, 75000)
Units: **undefined**

(10) FINAL TIME = 44 
Units: Week 
The final time for the simulation.

(11) Finding Delay=
2 
Units: **undefined**

(12) Halg=
IF THEN ELSE(Working Rate<0, 1, 0) 
Units: **undefined**

(13) IF of ED=
Alpha*(Retail Order*1+Information*0)+Expected Demand*(1-Alpha) 
Units: Dmnl

(14) IF of IF=
DELAY FIXED(Manufacturing Ability*(2^Double)*(0.5^Halg)*(0^stop), 1, 0)
(15) If of PD =
Back log * 0.2
Units: **undefined**

(16) IF of SR =
DELAY FIXED (Periodical Sales * Repeat Rate, 1, 0)
Units: **undefined**

(17) IF of UC =
IF THEN ELSE (Under Construction = 0, Expansion, 0)
Units: **undefined**

(18) Information =
MAX (Total Demand - Phantom Demand - Inventory at Retail, 0)
Units: **undefined**

(19) INITIAL TIME = 0
Units: Week
The initial time for the simulation.

(20) Inventory at Factory = INTEG ( IF of IF - OF of IF,
37500)
Units: **undefined**

(21) Inventory at Retail = INTEG ( OF of IF - Periodical Sales,
75000)
Units: **undefined** [0, ?]

(22) Investment Policy =
3
Units: **undefined**
(23) Manufacturing Ability = INTEG (Complete OF of MA, 37500) 
Units: **undefined**

(24) OF of ED = Expected Demand 
Units: **undefined**

(25) OF of IF = MIN(Inventory at Factory, Accumalated Order) 
Units: Dmnl [0,?] 

(26) OF of MA = IF of IF/160 
Units: **undefined**

(27) OF of PD = DELAY FIXED(If of PD, Finding Delay, 0) 
Units: **undefined** [0,?] 

(28) OF of SR = DELAY FIXED(IF of SR, 1, 0) 
Units: **undefined** [0,?] 

(29) OF of UL = IF THEN ELSE(Upper Limit>Diffusion Level, Back log*0.1, 0) 
Units: **undefined** [0,?] 

(30) Past Max = DELAY FIXED(Expected Demand, 1, 0) 
Units: **undefined**

(31) Periodical Demand = A*Diffusion Level*(Upper Limit-Diffusion Level) 
Units: **undefined** [0,?]
(32) Periodical diffusion=
Sales for Repeater-Periodical Sales
Units: **undefined** [0,?] 

(33) Periodical Sales=
MIN(Inventory at Retail, (Total Demand-Phantom Demand) )
Units: Dmnl [0,?] 

(34) Phantom Demand= INTEG ( 
If of PD-OF of PD, 0)
Units: **undefined** [0,?] 

(35) Received=
DELAY FIXED(Retail Order-OF of IF, 4,0)
Units: **undefined** [0,?] 

(36) Repeat Rate=
0.05 
Units: **undefined** 

(37) Retail Order= 
MAX(Total Demand-Inventory at Retail, 0)
Units: Dmnl [0,?] 

(38) Sales for Repeater= INTEG ( 
IF of SR-OF of SR, 0)
Units: **undefined** [0,?] 

(39) SAVEPER =
   TIME STEP
Units: Week [0,?] 
The frequency with which output is stored.
APPENDIX A. EQUATIONS USED IN THE TAMAGOTCHI™ SUPPLY CHAIN MODEL

(40) stop=
IF THEN ELSE(Working Rate<-3, 1 , 0 )
Units: **undefined**

(41) TIME STEP = 1
Units: Week [0,?]
The time step for the simulation.

(42) Total Demand=
MAX(Periodical Demand+Phantom Demand+Sales for Repeater, 0)
Units: **undefined** [0,?]

(43) Total Sales= INTEG ( Periodical Sales,
0)
Units: **undefined** [0,?]

(44) Under Construction= INTEG ( IF of UC-Complete,
0)
Units: **undefined**

(45) Upper Limit= INTEG ( -OF of UL,
1.25e+008*0.15)
Units: **undefined** [0,?]

(46) Upward=
IF THEN ELSE(Expected Demand>Past Max, 1 , 0 )
Units: **undefined**

(47) Working Rate=
(Expected Demand-Inventory at Factory)/Manufacturing Ability
Units: **undefined**
Appendix B

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCN</td>
<td>Supply Chain Network</td>
</tr>
<tr>
<td>CPFR</td>
<td>Collaborative Planning, Forecasting and Replenishment</td>
</tr>
<tr>
<td>SCOR</td>
<td>Supply Chain Operations Reference model</td>
</tr>
<tr>
<td>GSCF</td>
<td>Global Supply Chain Forum</td>
</tr>
<tr>
<td>CNN</td>
<td>Cellular Neural Networks</td>
</tr>
<tr>
<td>SCM</td>
<td>Supply Chain Management</td>
</tr>
<tr>
<td>SAP</td>
<td>System Analysis and Program Development</td>
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<tr>
<td>IT</td>
<td>Information Technology</td>
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<tr>
<td>VoA</td>
<td>Volkswagen of America</td>
</tr>
<tr>
<td>ERP</td>
<td>Enterprize Resource Planning</td>
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<tr>
<td>ICH</td>
<td>Inventory Collaboration Hub</td>
</tr>
<tr>
<td>EWM</td>
<td>Extended Warehouse Management</td>
</tr>
<tr>
<td>TMS</td>
<td>Transportation Management Systems</td>
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<tr>
<td>SCE</td>
<td>Supply Chain Execution</td>
</tr>
<tr>
<td>IMS</td>
<td>Inventory Management Systems</td>
</tr>
<tr>
<td>APICS</td>
<td>The Association for Operations Management</td>
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<tr>
<td>MRO</td>
<td>Maintenance, Repair, and Operations</td>
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<tr>
<td>CSCMP</td>
<td>The Council of Supply Chain management professionals</td>
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<tr>
<td>CRP</td>
<td>continuous replenishment process</td>
</tr>
<tr>
<td>VICS</td>
<td>Voluntary Inter-industry Commerce Standards</td>
</tr>
<tr>
<td>3PL</td>
<td>Third party logistics service providers</td>
</tr>
<tr>
<td>4PL</td>
<td>Fourth party logistics service providers</td>
</tr>
<tr>
<td>FGI</td>
<td>Finished goods inventory</td>
</tr>
<tr>
<td>WIP</td>
<td>Work in process</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
<td>--------------------------------------------------</td>
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<tr>
<td>CIMS</td>
<td>computer-integrated manufacturing systems</td>
</tr>
<tr>
<td>FA</td>
<td>factory automation</td>
</tr>
<tr>
<td>WMS</td>
<td>warehousing management systems</td>
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<tr>
<td>GIS</td>
<td>geographic information systems</td>
</tr>
<tr>
<td>SCC</td>
<td>Supply-Chain Council</td>
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<tr>
<td>DES</td>
<td>Discrete Event Simulation</td>
</tr>
<tr>
<td>NP-hard</td>
<td>Non-deterministic Polynomial-time hard</td>
</tr>
<tr>
<td>GSCM</td>
<td>Global Supply Chain Model</td>
</tr>
<tr>
<td>MIP</td>
<td>mixed-integer programming</td>
</tr>
<tr>
<td>SC</td>
<td>Supply Chain</td>
</tr>
<tr>
<td>ODE</td>
<td>ordinary differential equations</td>
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<tr>
<td>LTE</td>
<td>linear time invariant</td>
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<tr>
<td>VSC</td>
<td>variable structure control</td>
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<tr>
<td>PID</td>
<td>proportional + integral + derivative</td>
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<tr>
<td>TPS</td>
<td>Toyota Production System</td>
</tr>
<tr>
<td>SCE</td>
<td>supply chain execution</td>
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<tr>
<td>KPI</td>
<td>key performance indicator</td>
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<tr>
<td>AI</td>
<td>artificial intelligence</td>
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<tr>
<td>DSP</td>
<td>Digital Signal Processor</td>
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<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
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Bibliography


