Towards a Game-theoretic Model of Co-operative Context-aware Driving under Random Influences

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***Abstract*— Cooperative driving is currently an active area of research, where considerable effort is put on providing drivers with precise information about their surrounding and actions of others in order to have a valuable decision support system. Still, the final decision does and should always remain with the driver. Hence, any recommendation may be followed or disregarded by the driver, yielding to avoidance or increase of danger for others and oneself. This paper makes an attempt to model cooperative driving as a non-cooperative game with an explicit account for the free will of a driver. We derive results on how much the optimal situation may deviate from the actual situation if the driver is unwilling or unable to precisely follow the recommendation of a driving assistance system.**

I. Introduction

Various kinds of driving assistance systems (DAS) exist and offer support to avoid potentially dangerous situations. Communication among such systems can improve performance through extended/shared sensing capabilities. Humans make their decisions based on the perceived driving context, so it is natural to extend this approach to advanced driving assistance systems (ADAS). We shall not burden ourselves with legal issues here, and restrict the considerations to the (presently true) assumption that the final decision remains with the driver. This means that we may also expect negative consequences if the driver is unwilling or unable to follow the optimal decision as given by the DAS. Unwillingness may stem from overestimation of one's own capabilities or sufficient experience in order not to necessarily rely on an DAS's recommendation. Inability to follow a recommendation may be due to bad road or vehicle conditions, fatigue or bad health conditions, or similar.

We present a game-theoretic model that accounts for random choices of a driver that are "centered" around the optimal choice recommended by the DAS. More precisely, we assume that a driver is acting optimal in the long run average, but occasionally deviates from the optimal behavior for some reason. After briefly arguing the importance of context information, we describe the model and the theoretic framework of game theory for convenience of the reader. The paper continues with a result on how much deviation from the optimal situation is to be expected under randomly perturbed actions, and closes with a discussion of the approach.

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II. Related Work

The field of cooperative driving has been a very active research area over the last decade [1], [2], [3]. However, we are not aware of any proposal for estimating the "loss of safety" due to (un)intended misbehavior. To the best of our knowledge, this is the first model that can be used to explicitly measure deviations from optimal situations arising from misbehavior. A recent attempt to model driving behavior under extreme conditions, presented in [4], is not game theoretic, and as such essentially different from ours. The idea of using game theory for describing driver's behavior is not new, though:

The authors of [5] use extensive form games to model the driving process, but do not consider cooperation.

Dynamic driving games are used in [6] and [7], but there is no analysis of possible distortions.

The use of game theory to support and analyze the driving process is further backed up by the work of [8], who identifies ADAS as a valuable asset especially for older drivers. Actions chosen by kids or old persons may occasionally appear irrational or even random, because the person may either be unaware of a risk (kids), or too slow to react appropriately on certain incidents. Each is a source of possible danger and deserves closer investigation, which we provide a starting point for in this work.

III. A Context Model

The context mainly determines the decisions drivers are making, and we assume a context to be composed of four major components: 1) the environment, 2) the driver, 3) the own-vehicle, and 4) the driving regulations (traffic law). Subdivisions of each component are imaginable, such as the environment can be decomposed into the spatial environment (e.g. type of the road) and local environment (regional physical environment with specialized driving rules).

Information about the driver includes experience, the current state and the risk willingness. Also, recommended behavior and deviations from the true actions of a driver can be recorded and later be used for estimating the utility (safety) of certain actions.

In cooperative driving, we assume that DAS are continuously communicating and exchanging their perceived environments. Information about the quality of the data can be passed along. Deviations from the optimal behavior may not only be due to unwillingness or inability to follow, but also due to imprecise context information. Our model can theoretically account for this too, which is subject of future work.

IV. Game-Theoretic Model

We consider a finite set of drivers (players), denoted as $N = \{1, 2, \ldots, n\}$. Throughout the remainder of this work, we use the terms *driver* and *player* synonymously. With the set of players N, we have a non-cooperative n-Person game in *normal form* being a triple $\Gamma = (N, S, H)$, with $S = \{S_1, S_2, \ldots, S_n\}, S_i \subset \mathbf{R}^d$ being a set of compact and bounded d-dimensional *strategy sets*, and $H = \{u_1, \ldots, u_n\}$ being a set of *payoff functions* dependent on the strategies of each player, i.e. $u_i : \times_{i=1}^{n} S_i \to \mathbf{R}$ for all $i \in N$. We henceforth omit the term "non-cooperative" and simply speak of games henceforth. A pure strategy is any member of the strategy set of a player, as opposed to a mixed strategy, which is a probability distribution over the strategy set. We shall concentrate on mixed strategies, and their implied expected average, which we again denote as $u_i(s_i, s_{-i})$, with the notational convention that s_i is the mixed strategy chosen by the i-th player, and s_{-i} is the set of mixed strategies chosen by i's opponents.

Following the proposal of [9], we can define an appropriate utility function measuring safety of a driving process as follows: Assume that the ADAS continuously records differences between its recommendations and the true behavior of a driver. Together with the context model, we can set up simulation models for certain situations, which can then be used to run simulations under various alternative actions a driver can take. Knowing the driver (context model), we can simulate her/his random behavior and count the number of trials in which the simulated choice has yielded a successful maneuver. The relative frequency of the successful trials then converges to the probability of success, which is the utility function we use. As an example, consider a driver wishing to overtake another 4m long vehicle in front of him (both of which are driving at 100km/h with 5m distance), facing oncoming traffic (at 100km/h, 230m away) on the other direction lane on his left hand side. Knowing the speeds of the other vehicles (through communication among the ADASs), and knowing that the own speed is maintained with a variance 10, running 10000 simulations with (normally distributed) random choices of the optimal (recommended) overtaking speed $v \sim N(120km/h, \sigma^2 = 10)$, we get a relative frequency of ≈ 0.02 for an accident, and an according utility of $\approx 1-0.02 = 0.98$ for the optimal choice under random perturbation.

A Nash-equilibrium of a game is a situation in which each player chooses his best strategy upon knowing the strategies of all the other players. This knowledge is assumed to be available through the communication among the ADAS, so Nash's well-known existence result as well as its generalizations apply under this assumption.

In our context, we may summarize random deviations through "free will" or suboptimal road/vehicle conditions as *acting semi-rational*, since a driver may have a quite good idea what to do, but cannot exactly realize the strategy. This gives rise to the following definition:

Definition 1: (Game with semi-rational players) Let a game $\Gamma = (N, S, H)$ in normal form be given. A *non-cooperative game with semi-rational players* is a quadruple $\Gamma = (N, F, S, H)$, where $F = \{F_1, \ldots, F_n\}$ is a set of probability measures of random variables X_i over a metric space $\mathcal{S}$. Each random variable X_i satisfies $\mathbf{E}(X_i) = 0$; the set $\mathcal{S}$ is assumed to be a continuum and $S_i \subseteq \mathcal{S}$ for all strategy spaces S_i.

The independent random variables $X_1, \ldots, X_n$ reflect the errors each player inevitably makes when trying to realize a chosen strategy, i.e. if s_i^* is the optimal strategy for player i, then the strategy actually played is $s_i^* + X_i$, where $X_i \sim F_i$. The constraint $\mathbf{E}(X_i) = 0$ models the assumption that players still perform optimal in the long run average, but although a fully rational strategy is attempted, no player can advance the own skills beyond some personal/technical limit.

Nash's definition of an equilibrium is easily adapted to apply for our model: Formally, an equilibrium situation is present if $s^* = (s_i^*, s_{-i}^*)$ is a better strategy for driver i than any of the alternatives, i.e. $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall i, \forall s_i \in S_i$. Replacing the optimal strategy s^* by an imprecise strategy $s_i^* + X_i$, the terms become random, and we can ask for the long-run average, which is $\mathbf{E}_{X_i}(u_i(s_i^* + X_i, s_{-i}^* + X_{-i}))$. Accordingly, we define an equilibrium for a game with semi-rational players by being optimal in terms of expected utility:

Definition 2: (equilibrium for games with semi-rational players) Let $\Gamma = (N, F, S, H)$ be a non-cooperative game in normal form with semi-rational players, then a strategy s^* is called an *equilibrium*, if the expected utility of each player is optimal under the chosen strategy s_i^*, i.e. if $\forall i, \forall s_i \in S_i$,

$$\begin{aligned} &\mathbf{E}_{X_i}(u_i(s_i^* + X_i, s_{-i}^* + X_{-i})) \\ &\qquad \geq \mathbf{E}_{X_i}(u_i(s_i + X_i, s_{-i}^* + X_{-i})). \end{aligned} \quad (1)$$

There is a natural relation to the concept *correlated equilibria* [10], where players derive their strategies from correlated random variables. This, however, is not the case in our setting, since we assume randomness of strategies to be uncorrelated among players, but their choices to be correlated through the utility function. The latter is the case in any game, but unlike correlated equilibria, imprecisions of actions of a driver are independent of the deviations other drivers exhibit. The existence of equilibria is not instantly obvious, but can be established:

Theorem 4.1: Every game with semi-rational players with nonempty compact strategy spaces, continuous payoff functions and compactly supported uncertainty measures has at least one equilibrium. *Proof:* We start by proving a simple claim: *For every game with semi-rational players,* *there exists a normal form game with the same set of equilibria*. Define the random variable $Y_i := -X_i$, then Y_i has the density $g_i(x) := f_i(-x)$, by a change of variables [11]. With $s = (s_i, s_{-i})$, re-write the right-hand side of (1) as $\mathbf{E}(u_i(s_i^* + X_i, s_{-i}^* + X_{-i})) = \mathbf{E}(u_i(s_i^* - Y_i, s_{-i}^* - Y_{-i}))$ $= \mathbf{E}u_i(s - Y_i)$. By definition, the last term is

$$\mathbf{E}u_i(s - Y_i) = \int_{S_i} u_i(s-y) f_i(-y) dy = u_i * g_i.$$

Rewriting the equilibrium condition (1) in terms of $\widetilde{u}_i := u_i * g_i$ gives $\widetilde{u}_i(s_i^*, s_{-i}^*) \geq \widetilde{u}_i(s_i, s_{-i}^*) \quad \forall i, \forall s_i \in S_i$, which is the defining condition of Nash-equilibrium. It follows that a situation s^* is an equilibrium for a game $\Gamma = (N, F, S, H)$ with semi-rational players if and only if it is a Nash-equilibrium of the game $\Gamma' = (N, S, H')$ in normal form with $H' := \{u_i * g_i | g_i(x) = f_i(-x), f_i \in F\}$, which proves the claim.

The theorem now instantly follows from a generalization of Nash's result to games with continuous and compact action spaces. The assumption of compactly supported measures makes the resulting strategy spaces compact again, and the existence result of [12] establishes the theorem. ∎

Having the existence of equilibria situations, we can ask for how much those equilibria deviate from a situation that would arise in a game with perfectly precise actions. This is dealt with in the next section.

V. Quantifying Deviations

We would like to obtain some measure giving the deviation of Nash-equilibria when moving from exact play to imprecise strategies. Let a game with semi-rational players be given, then call $\Gamma(F) = (N, S, H)$ the "standard" game as constructed in the proof of theorem 4.1. Without loss of generality, we henceforth refer to this game in our further considerations.

The goal of this section is a bound on the loss of utility by semi-rational actions.

Theorem 5.1: Let Γ be a game with semi-rational players. If the payoff functions are continuous and convex in each strategy, and the strategy spaces are compact, then for any $\varepsilon > 0$, the probability of deviating more than ε from the optimal utility is exponentially small. More precisely,

$$\begin{aligned} \Pr\left\{\left|u_i(s_i^*, s_{-i}^*) - u_i(s_i^* + X_i, s_{-i}^*)\right| \leq \varepsilon\right\} \\ \geq 1 - 2\exp\left(\frac{-\varepsilon^2}{n\left\|u_i\right\|_\infty^2}\right), \end{aligned}$$

for $s_i + X_i$ being the randomly perturbed strategy of the i-th player.

Proof: Let $s^* = (s_1^*, \ldots, s_n^*)$ be an equilibrium situation according to definition 2. Call $Y_i := s_i^* + X_i$ the random variable that reflects player i's behavior. By the convexity of u and Jensen's inequality [13], $\mathbf{E}u_i(Y_i, s_{-i}^*) \geq u_i(\mathbf{E}Y_i, s_{-i}^*) = u_i(s_i^*, s_{-i}^*) \geq u_i(Y_i, s_{-i}^*)$, where the second inequality follows from the definition of an equilibrium. Notice that within the chain of inequalities, the optimal utility $u_i(s_i^*, s_{-i}^*)$ lies somewhere between $\mathbf{E}u_i(Y_i, s_{-i}^*)$ and $u_i(Y_i, s_{-i}^*)$. This difference (and hence the difference between the optimal and actual utility) can be bounded using McDiarmid's inequality [14, Lemma 1.2] as follows: Since u_i is continuous on a compact set, we have $\|u_i\|_\infty < \infty$, and by McDiarmid's inequality, we can bound the probability of the above difference to exceed some value $\varepsilon > 0$ by

$$\Pr\left\{\left|u_i(s_i^*, s_{-i}^*) - u_i(Y_i, s_{-i}^*)\right| > \varepsilon\right\} \leq 2\exp\left(\frac{-\varepsilon^2}{n\left\|u_i\right\|_\infty^2}\right),$$

which completes the proof. ∎

Increased speed is often not necessarily tied to decreased travel time. First, we may not legally increase speed over the allowed limit, and secondly we still have to pay attention to the other driver's actions. Conversely, if one driver increases the speed and is copied by the others, then we are back at the situation where drivers follow similar speed patterns, in which case nobody can significantly win over another. This intuitively indicates that the travel time has a minimum somewhere in the middle, and the assumption of convex utility intuitively makes sense for this example.

Theorem 5.1 can be interpreted so that the probability of large deviations is exponentially small, i.e. sudden abrupt effects are unlikely to occur even under partially random behavior.

VI. Conclusion

We have presented a game-theoretic model for cooperative driving that explicitly accounts for the freedom of disregarding the recommendations negotiated by advanced driving assistance systems. Our model assumes each vehicle to be equipped with an advanced driving assistance system, which provides recommendations that are optimal in the sense of having highest probability of successful completion of maneuvers. Direct communication and cooperation is assumed to happen mostly among the ADAS, which present their recommendations to their respective drivers. We derived results quantifying the expected loss of safety (in a probabilistic sense) under random deviations from optimal actions, due to influences that may be beyond the drivers control, due to missing information, or due to the drivers intentional disregard of a recommendation. From our analysis, we may conclude that small uncertainties in the behavior of drivers are unlikely to yield severe safety problems. Stated differently, we have shown that the probability of large deviations from optimal situations is exponentially small.

Our model is general, and as such applicable to a wide range of applications, not necessarily limited to the analysis of road traffic. Future work in this direction includes relaxing the assumptions of theorem 5.1, deriving deterministic rather than probabilistic bounds, and analyzing the global view of the collective (as opposed to the local view of the driver which has been analyzed here). We believe in the potential of our results to act as a valuable extension for the analysis of cooperative driving. Analyzing the global view and the loss of social welfare under simultaneous random behavior of participants is subject of future research.

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