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BIFURCATION ANALYSIS AND SYNCHRONIZATION ISSUES IN A THREE ECHELON SUPPLY CHAIN NETWORK

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Abstract

This paper introduces the importance of integration in supply chain network (SCN) and briefly presents the importance of synchronization in SCN. The evaluation of the causes of uncertainties within the supply chain networks is considered. The structure of a three- echelon supply chain is envisaged and the modeling of this structure is carried out. It is shown that the supply chain network can be exposed to both external and internal perturbations. The origin of these perturbations is discussed and it is shown that the stability of the supply chain network is very sensitive to these perturbations. The effects of the external perturbations on the supply chain network are considered. A method is developed which is based on an adaptive algorithm for the automatic cancellation of the effects of external perturbations by re-adjusting the internal thresholds of the supply chain network in order to achieve synchronization. The bifurcation analysis is presented for the chosen scenario as it helps to discover the various states towards the achievement of synchronization.

Keywords: Synchronization; Supply chain networks; Supply chain network control; Supply chain optimization; Bullwhip effect; Chaos in supply chains, bifurcation analysis in supply chains.

Introduction

Supply chain management (SCM) is the combination of management and science. The modern supply chain networks are global in nature and have many interconnections i.e. dependency or coupling between each. The primary objective of the supply chain network, or the individual entities in the network, is to deliver the product in the correct quantities at correct time by meeting all the quality measures with competitive price. In addition to this, the threshold of stocked products, quantities must be continuously sustained and uncertainties (that can occur within the supply chain) must be well controlled in order to avoid breaking in stocks and also the well known bullwhip effect (Chen, 2000, Dejonckheere, 2003) at a given level of the supply chain. This can be achieved through a good coordination or synchronization between the various levels of supply chain. In order to achieve the stated objective, all the stakeholders in the supply chain network are integrated/coupled (. The first part of coupling is concerned with functional integration of purchasing, manufacturing, transportation, and warehousing activities.

Even though integration of the stakeholders should provide the ideal solution for the optimization of the supply chain networks, in many cases it is not. Business is not usual always; there are many things that happen with time. The major source for uncertainties in supply chain networks originates from the fact that operations are performed over long period of time and stakeholders are dispersed globally (Wilding,1998). Supply chain management is a complex set of processes and flows (information flow, product flow, monetary flow) that can either add a great deal of efficiency and profit or can cause serious problems for a company. SCM is technology intensive in some aspects, in some ways it can be compared to a simple game gossip game. The supply chain can often be seen as the same circle. Today's supply chains pass information relating to item information, forecasting and replenishment upstream and downstream among manufacturers, brokers, distributors, retailers and warehouse, transportation, management and global trade systems. The final result is a flow of information from the creation of a product to the point where that product is sold to the end consumer. As in the gossip game, the accuracy of information often depends on the quality of the data at the beginning of the cycle, and maintaining same information throughout the supply chain. If the information is wrong in the beginning, it never has a chance of being correct when it finally gets up or down the supply chain. In fact, if inconsistencies remain, there are chances for errors to propagate through the whole supply chain (Hwarng, 2006).

The definition of a truly efficient supply chain is when all companies involved are communicating correct or reliable data. However communicating the reliable data is always not possible in supply chains as each stake holder has different objectives and constrains. The goal of supply chain management is to optimize the whole system (Sterman, 1989). Total chain-wide transportation, holding, and setup costs should be minimized. However, this integration may be difficult to achieve because different members of the chain have conflicting objectives (Khouja, 2003). All these leads to more inventory at different levels and can cause the well known bullwhip effect or forester effect. In this paper, the occurrence of this effect and also the chaotic effect in the supply chain due to internal perturbation, external perturbation and both internal and external perturbation of the system is shown.

Supply Chain Modeling

The theoretical framework for the supply chain management underlies the setting, optimization and control of the system model. Here, In this work we use the structure of a three-level supply chain as illustrated in Fig. 1.

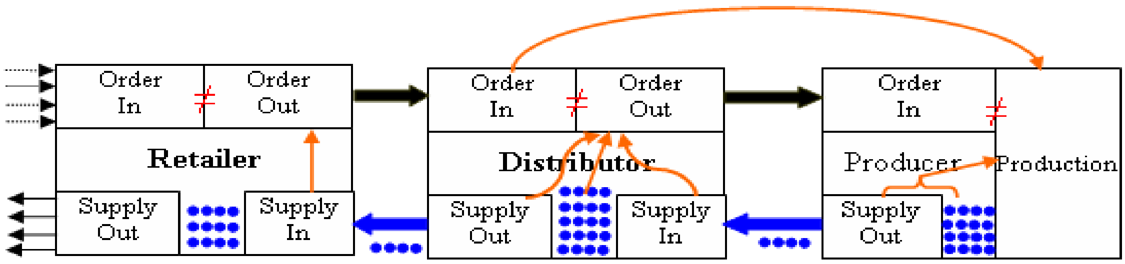

Figure 1: Three level supply chain model

A three echelon model is envisaged in Figure 1 to describe a simple scenario in a very complex supply chain. Figure 1 also illustrates the relationship between the three layers. The orders they make may not be equal to orders they receive. The order out quantity depends not only on how much inventory you have already, and how much you want to supply out. Order out quantity at retailer depends on the ratio m at which the demand is satisfied during the previous order. Distributor needs to take into consideration among other things, the distortion rate r that can occur in the inventories. The producer needs to take care about the safety stock k in order to avoid the small production batches. These phenomena are explained below with mathematical modeling.

We consider that the demand information is transmitted within the layers of the supply chain with a delay of one unit time. As illustrated in Fig. 1, the ordering quantity is not same as the requested order quantity at any level. The order quantity at current period of time at retailer is linearly coupled with the distributor and it is influenced by how much of demand is satisfied in previous period of time. This phenomenon can be represented as shown in Eq.1.

$$x_i = m(y_{i-1} - x_{i-1}) \tag{1}$$

m is the ratio at which the demand is satisfied. As it appears from Fig. 1, the dependency or coupling of distributor on producer and retailer is no more linear. Indeed the distributor needs to take the combined effect of retailer and producer into consideration before making his order, i.e., quadratic coupling. Apart from this, the distributor also needs to take into consideration the expected loss rate or distortion that can take place on the producer's supplies. This can be expressed mathematically as shown in Eq. 2.

$$y_i = x_{i-1}(r - z_{i-1}) \tag{2}$$

r is the distortion coefficient. The production quantity from the producer unit typically depends on the distributor's orders and the safety stock. However the distributors' orders again depend on the retailer's orders, i.e., the producer needs to take the combined effect of retailer and distributor into account before making production decisions. This can be represented as

$$z_i = x_{i-1}y_{i-1} + kz_{i-1} \tag{3}$$

Eqs. 1-3 represent the quantity demanded by customers (Eq. 1), the inventory level of distributors (Eq. 2) and the quantity produced by producers (Eq. 3).

- Where $x_i < 0$ denotes that the supply is less than customers demand in the previous period

- y_i <0 denotes that information is severely distorted and no adjustment is necessary to inventory level
- z_i <0 denotes the cases of overstock or return and hence no new productions.

Figure 2: Reference System with given coefficients
Evidently the continuous forms of Eqs 1-3 can be rewritten as *Lorenz* equations of state:

$$\dot{x} = \sigma(y - x) \tag{4}$$

$$\dot{y} = rx - y - xz \tag{5}$$

$$\dot{z} = xy - bz \tag{6}$$

Depending upon the parameter values, this model produces a wide variety of nonlinear features. In Figure 2, we showed the reference model with the parameter values $\sigma = 15$ $r = 29$, and $b = 2/3$ to illustrate the regular state of the reference supply chain model.

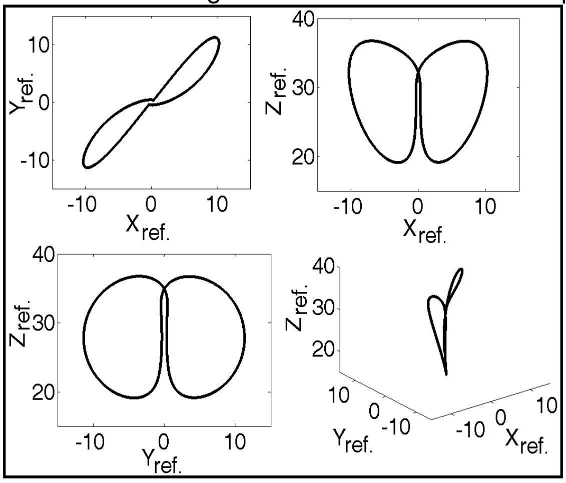

Figure 2 Reference System with given coefficients

In this work, we consider both internal and external perturbation of the supply chain network. The internal perturbation, which is linear, is in large part due to the internal structure of the system. We consider the external perturbation as a nonlinear perturbation which is in general from the out side system. Assume the equations of state of Eqs 4-6 in perturbation as follows by considering both internal perturbation and external perturbation

$$\dot{x}' = (\sigma + d\sigma)(y' - x') + d_1 \tag{7}$$

$$\dot{y}' = (r + dr)x' - y' - x'z' + d_2 \tag{8}$$

$$\dot{z}' = x'y' - (b + db)z' + d_3 \tag{9}$$

Where $d\sigma$, dr , db represents the internal perturbations and $d_i\,(i = 1,2,3)$ represents the external perturbations.

Synchronization in supply chain networks

The modern synchronization tools provide automation of the paper work but not concentrate on what happen if the given data is slightly changed accidentally. Effective data synchronization first involves making information visible within a secured environment, which is again a case that we are discussing throughout this paper. In this context we provide different cases of perturbation to the data and present the synchronization for the system in chaos. The causes of supply chain instability can be broadly classified into two categories. The first cause is the dynamical and nonlinear character of the motions (i.e. material/products flow, information exchanges, etc…) between different entities in supply chains. The second cause originates from the effects of both external and internal perturbations (Lei, 2006) to which the supply chain is subjected. An optimal management of the information flows within the supply chains may be of high importance in order to alleviate the effects resulting in negative consequences on the flows within the supply chains. This could be achieved through an adaptive control mechanism which is based on a current comparison of the dynamical data within the supply chains with the pre-defined data fixed by the requirements of the supply chains. Here, an automatic or adaptive control of the flows within the supply chains should be able to detect changes in the flows within the supply chains and act accordingly/consequently (by undertaking a given action within the supply chains) in order to alleviate the undesirable effects and therefore stabilize the system behavior that has been perturbed. The achievement of synchronization is observed when the action undertaken has allowed the recovery of the original behavior (eventually thresholds or reference requirements) of the supply chain.

This paper develops an adaptive method (algorithms and/or tools) for the systematic and automatic control of the flows within the supply chains. The external perturbations can be a consequence of the dynamical behavior of the market demand, forecasting methods, high lead times, etc. In fact, due to the dynamic changes as discussed (in time domain), some pre-defined data or requirements within the supply chains (thresholds like safety stocks) may be varying accordingly as consequence of these perturbations. The internal perturbations within the supply chains are random (or undesirable) effects which can originate from the malfunctioning of a given echelon of the supply chain (e.g. Machine breakdowns, transportation delays etc.,). It should be worth mentioning that a combination of the simultaneous effects of both internal and external perturbations may be responsible of the dynamic motion variations (e.g. flow of materials, information exchange, etc.) within the supply chain. This is a concrete and/or realistic scenario as the supply chains of many companies are currently exposed to the both types of perturbations.

The external excitation can be better explained by the fluctuations or instability of the market demand. The effects of these fluctuations on the reference model of the supply chain are significant due to the extreme sensitivity of the supply chain global requirements (e.g. data flows, materials flows, etc..) to tiny changes (i.e. external perturbations or internal perturbations).

The analysis in this work is restricted to the case where the reference model is subjected to external perturbations due to the fluctuation in the market demand. We assume the case where the reference model is externally perturbed by periodic fluctuations in the market. Figure 3 represents the attractors of the reference or original supply chain under external perturbations. The effects of external perturbations on the original or specific requirements of the reference supply chain are clearly shown on this figure by the well-known chaotic Lorenz attractors.

The cancellation process of these effects is demonstrated and its achievement is through synchronization. Indeed, due to the external perturbation of the reference supply chain, we show the possibility of undertaking an internal action within the supply chain in order to annihilate the effects due to external perturbations. The internal fluctuations are represented by $(d\sigma, \ dr, \ db)$. To alleviate the effects due to external perturbations, we develop a numerical code based on a regulation feedback. In fact, the numerical code considers the effects of external perturbation and adjust the values of the internal parameters of the three echelon supply chain by incrementing all of them simultaneously whereby the incrementing of each internal parameter is performed in a well defined windows or interval of variation. The numerical code always compares the results from the supply chain submitted to both external perturbation and the internal adjustment of the system parameters with the results of the reference (or original) supply chain behavior. A threshold error was fixed (which is less than approximately 0.1) under which full alleviation of the effects due to external perturbation is supposed to be effective, leading to the achievement of synchronization, which results in the recovery of the reference system behavior. Figure 4 represents the attractors of the reference or original supply chain subjected to both the action of external perturbations and some fluctuations of the internal values. The regulation process has been exploited to determine the following values $(d\sigma = 35, \ dr = 15, \ db = 0.09)$ of internal fluctuations for which the synchronization is achieved. This can be shown on Figure 4, which shows attractors similar to those of the reference or original supply chain system.

Bifurcation Analysis

The preceding section has shown that the complete annihilation of the effects due to external perturbations is possible through the achievement of synchronization. Nevertheless, during the regulation process, we found that the perturbed supply chain system is very sensitive to internal fluctuations. Indeed, it was observed that the motions in the global supply chain are submitted to various states raging from regular to chaotic states. These regular and/or chaotic states were observed when monitoring the internal parameters (i.e. $0 \le d\sigma \le 50$ and $0 \le dr \le 50$) of the supply chain. Therefore, the bifurcation analysis can help to discover the various states towards the achievement of synchronization or the complete annihilation of the effects due to external perturbations.

Figure 5 is the plot of the bifurcation diagram showing the extreme sensitivity of the perturbed supply chain to internal variations i.e. $0 \leq d\sigma \leq 50$. This windows/range of (variation of) the internal parameter is used to show that the step towards synchronization is through regular and chaotic states. Figure D reveals that windows of regular motions alternate with windows of chaotic motions e.g. period-1, period-3, and chaotic attractors are shown. The plot of bifurcation diagrams is of necessary importance as it can help to know the range of the internal parameters in which the synchronization can be achieved. For instance, considering the periodicity of the original supply chain (Figure-A) i.e. period-1 attractor, synchronization (or the annihilation/cancellation of the effects due to external perturbations) is achieved if the same periodicity is obtained as the result of the competition between the internal variation of the system values and the effects of external perturbations. This condition is one of the various conditions (say additional conditions are define in our numerical code for training process) to be fulfilled for the achievement of synchronization. Therefore it can be deduced from Figure 5 the following windows (i.e. $0 \leq d\sigma \leq 50$, and $30 \leq d\sigma \leq 50$) of the internal variations in which the synchronization phenomenon can be achieved. It should be worth noticing that this criterion was exploited to define the ranges or windows of internal parameters in which the regulation process was perform. This helped to avoid performing in the windows in which no synchronization is possible. The same scanning process was performed in Figure 6 where the plot of bifurcation diagrams is presented for the following variations of the internal parameter $0 \leq dr \leq 50$. This figure also confirms that the step towards synchronization is through the exhibition of regular and chaotic states. It is obtained from Figure 6 that the synchronization phenomenon can be achieved in the following range $0 \leq dr \leq 27$

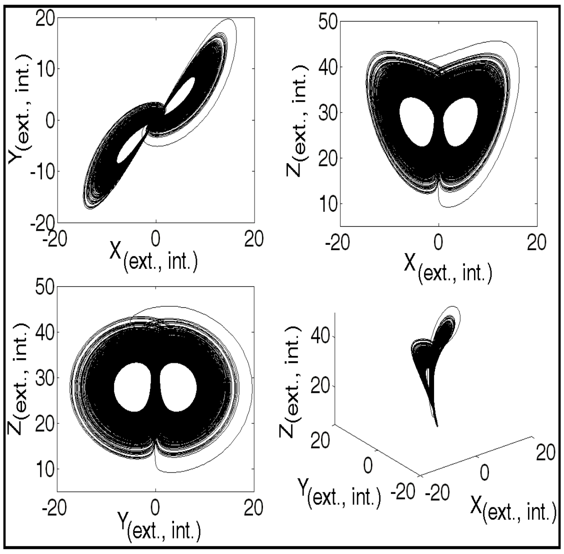

Figure 3 Chaotic state of the supply chain due to external perturbations

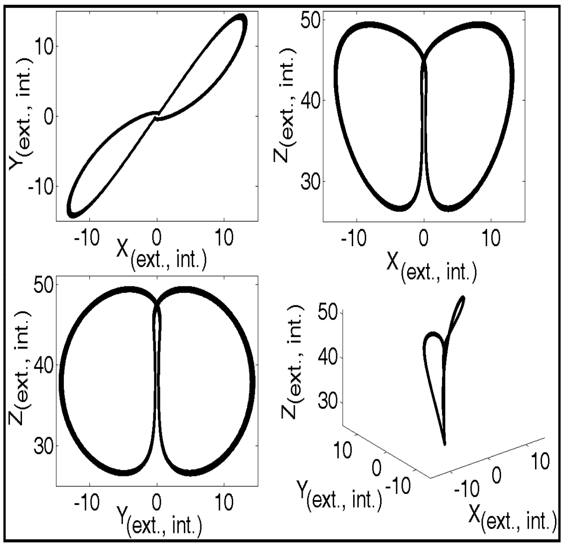

Figure 4 Alleviation of the chaotic effect caused by external perturbation with adaptive numerical code

Conclusion

The management of supply chain networks is a complex issue which involves numerous dynamic situations varying with time. This work illustrates how a supply chain can go to chaotic mode causing forester effect, if the parameters of the external perturbations are changed. The states illustrated in this work can occur to different parameter values i.e., they are not specific to only a set of values. This work also includes the bifurcation analysis of the system with in a window/range for two different internal parameters. This analysis helps in better understanding of the whole SCN in certain range. This paper also proposes an adaptive algorithm for the automatic cancellation of the effects of the external perturbations by re-adjusting the internal thresholds. This method is particularly appealing as it is possible to control or adjust the internal thresholds of the supply chain network. The solutions proposed in this paper offer a new range of possibilities for risk managers and provide a future research direction. The future research is also proposed in arriving at the adaptive synchronization with the help of analog simulation based on CNN (Cellular Neural Network) technology. An interesting and challenging problem of practical interest may be the development of adaptive numerical codes to achieve synchronization within a supply chain network under the

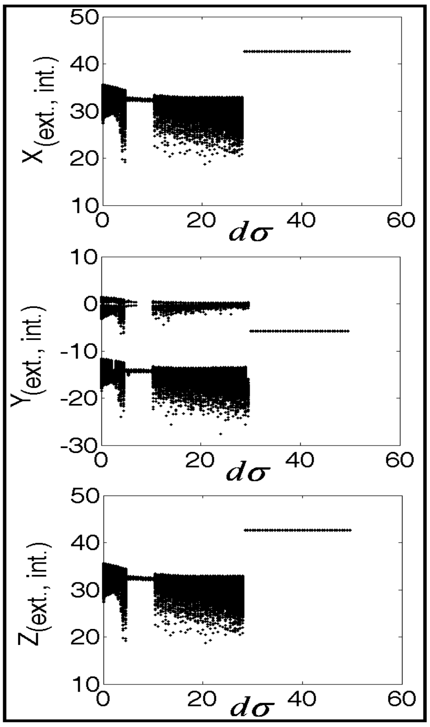

Figure 5 Bifurcation plot showing the sensitivity of the SCN to the internal variable $d\sigma$

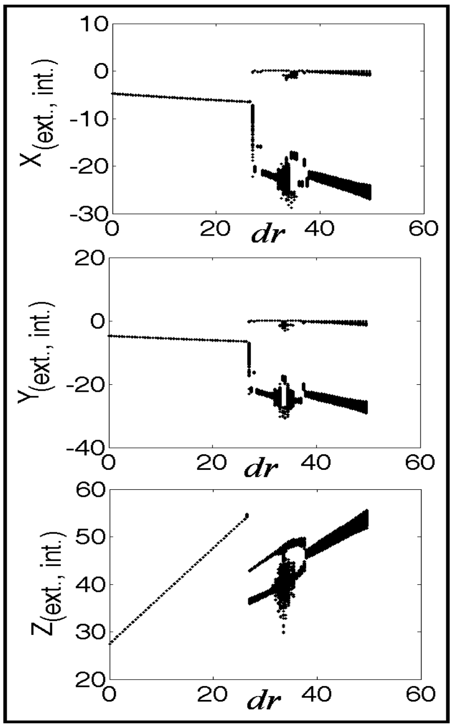

Figure 6 Bifurcation plot showing the sensitivity of the SCN to the internal variable dr

cumulative effects of both internal and external perturbations. This is a realistic scenario which currently manifests itself in commercial supply chain networks.

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